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A redetermination of the constant of precession, the motion of the equinox and the rotation of the galaxy from faint stars observed at the McCormick Observatory,

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Summary. From the proper motions of faint stars recently determined at the McCormick Observatory a new determination has been made of the constant of precession, the correction to NEWCOMB's motion of the equinox and the constant B of galactic rotation. No other unknowns were introduced in this solution, the position of the apex being inferred from bright stars, and the secular parallaxes from a recent study of the random motions of faint stars. In the treatment of the McCormick motions by VAN DE KAMP and VYSSOTSKY these quantities were considered as unknowns to be determined from the same material. In particular they also introduced a systematic difference between the parallaxes on the two sides of the galactic plane; the reality of the considerable difference found by them is doubtful, however, and might well have caused a systematic error in the determination of the precession. However, the new results for the three constants appear to deviate only slightly from those found by VAN DE KAMP and VYSSOTSKY.

A general comparison has been added of various determinations of the constants of precession and rotation. There is a discrepancy between the direct determination of the motion of the equinox and that found from proper motions. A tentative combined solution yields a correction of $+''^{/a}.0089 \pm ''^{/a}.0010$ (p.e.) to NEWCOMB's constant of lunisolar precession, and a correction of $+''^{/a}.0085 \pm ''^{/a}.0010$ (p.e.) to Boss's motions in right ascension on account of an error of NEWCOMB's motion of the equinox.

The average residuals shown in Tables 3*a* and 3*b* indicate that the large scale systematic errors of the fundamental system used are satisfactorily small, but the data do not yet appear sufficiently accurate for an adequate investigation of these errors.

Recently VAN DE KAMP and VYSSOTSKY have published interesting discussions based on a very fine and extensive material of proper motions collected by them at the McCormick Observatory[1]). These motions, which are of faint stars down to about 12.4 photovisual magnitude, have been derived photographically by referring them to stars of Boss's *Preliminary General Catalogue*, later supplemented by some stars of the still unpublished new *General Catalogue* of the Dudley Observatory, these Boss stars having as a rule been artificially reduced to the tenth magnitude. The motions of the faint stars found are on the system of the new *General Catalogue*, as the motions of this catalogue were used in the reduction[2]).

VAN DE KAMP and VYSSOTSKY have used these proper motions to determine the solar apex, the mean secular parallaxes, the constant of precession, the systematic error of the motion of the equinox and the constants of galactic rotation. Separate determinations were made for the secular parallaxes in different zones of galactic latitude, north as well as south of the galactic plane. The results deviated in several respects from what had been commonly assumed, the most apparent effect being perhaps a systematic difference amounting to about 30 % between the parallaxes north and south of the galactic plane. The authors considered this result as significant, and to some extent it might be supported by the good interagreement of the positions for the apex as derived from different galactic zones. However, in my opinion the reality of the result is extremely doubtful. Mean parallaxes derived from the distribution of the proper motions show no indication of it, and, moreover, it is not reflected in the star counts. From the mean parallaxes of Table 9, *B.A.N.* No. 290, the ratio of the mean parallax for Areas north of the galactic plane to that for Areas south of it was found to be 1.04 for $m_{pg} = 12.0$, .89 for $m_{pg} = 13.0$, and 1.02 for

1) *Astronomical Journal*, **45**, 161, 1936.
2) For details compare VAN DE KAMP and VYSSOTSKY, l.c. p. 162.

$m_{pg} = 14{\cdot}0$, or $\cdot 95 \pm \cdot 04$ (p.e.) for the average of the three magnitudes. The Areas between $+60°$ and $+90°$ latitude were excluded because no corresponding southern Areas were available, and the southern Areas were so adjusted that the distribution of the absolute values of the latitude was identical for the two hemispheres. In a similar manner an average ratio of $\cdot 99 \pm \cdot 05$ (p.e.) was found from the mean parallaxes derived by Mr VAN HOOF[1]). A comparison of the star counts in the general parts of the sky observed at McCormick shows down to 13·0 photographic magnitude an excess of only about 16 % for the southern galactic zones over the northern, which would correspond to a difference in mean parallax north minus south of about 6%. It is rather improbable that such a large difference in mean parallax as 30% would occur without a corresponding difference in numbers of stars per square degree.

For these reasons I am inclined to consider the difference found by VAN DE KAMP and VYSSOTSKY as spurious. The same remark and the same two argumentations as above may be applied to the rather abnormal variation of the McCormick secular parallaxes with average latitude, north as well as south of the galactic plane; they appear to vary in a non-monotonous way, showing a secondary maximum near the galactic circle, which the authors consider as real. The principal anomaly, however, lies in the fact, that, if the secondary variations are smoothed out, the McCormick results do not show any progressive increase from 0° up to about 50° latitude, whereas according to data derived from residual motions, as well as according to computations, they should increase with more than 100 % between 0° and 50°.

A closer inspection of VAN DE KAMP and VYSSOTSKY's Table 2.2[2]), from which these results were obtained, shows that the reality of the differences mentioned does not appear to be warranted by the accuracy of the determinations. To demonstrate this we may take the eight determinations given in the table for the zones below 60° latitude. Their simple average is 12·9. The average deviation from this mean is $\pm$ 2·2, while the average accidental error of one determination, as inferred from the probable errors given, is $\pm$ 1·6. The deviations are thus not measurably larger than the accidental errors to be expected. It is true that they look systematic, but the probability of finding five differences of the same sign for one of the three components of the solar motion is not small enough to attach a real significance to the difference.

As it might thus be feared that the introduction of the systematic difference of 30% between parallaxes north and south of the galactic plane had influenced VAN DE KAMP and VYSSOTSKY's solution of the precessional constants and the rotation of the galaxy, and as, moreover, accurate data for mean parallaxes and their variation with galactic latitude had become available through the investigation of the peculiar motions of the *Radcliffe Catalogue*[1]), it seemed of some interest to try a new solution, in which only those three quantities which cannot be sufficiently well determined from other data, namely the constant of precession, the correction to the motion of the equinox and the constant B of galactic rotation, were introduced as unknowns. So far as I know there is at present no other material available which is so excellently adapted for the determination of these constants as that collected at McCormick. The fact that the part of the sky south of $-25°$ is not covered is perhaps not so entirely disadvantageous as might be supposed, because just that region in which the systematic errors in the proper motions are likely to be most serious is thus excluded.

Through the great kindness of Dr VAN DE KAMP and Dr VYSSOTSKY the mean observed motions in galactic co-ordinates of stars in various intervals of photovisual magnitude were communicated to me in advance of their publication.

Two independent solutions were made, from the mean motions for the interval between photovisual magnitudes 10·5 and 11·4 (mean $11^m{\cdot}0$) and between 11·5 and 12·4 (mean $11^m{\cdot}9$) respectively. These motions were first corrected for the motion of the sun, and provisional corrections were applied in order to eliminate the main effect of the rotation of the galaxy.

The mean parallaxes needed for computing the reflex of the solar motion were derived with the aid of Table 13, *B.A.N.* No. 290 and the formula given just above that table. The photographic magnitudes corresponding to the mean photovisual magnitudes 11·0 and 11·9 used were obtained by adding a correction of $+0^m{\cdot}8$ for groups below 15° latitude and $+0^m{\cdot}7$ for those above 15°[2]).

For the motion of the sun the results derived from the bright stars in general might have been adopted, but as it is not impossible that this is influenced by local effects, it has seemed preferable to base the calculations for the faint stars upon the solar motion as determined from stars distributed over a large region of space, like the fainter O- and B-type stars observed at Victoria, or from types of stars in which no tendency to form groups is observed, like K- and M-stars. From 1800 G5—M-stars VAN HOOF[3]) has recently derived a

[1]) *B.A.N.* No. 289, Table 4.
[2]) L.c. p. 168.

[1]) *B.A.N.* Nos. 289 and 290.
[2]) C.f. Table 16, *B.A.N.* No. 290.
[3]) *B.A.N.* No. 269, 1935.

solar motion of 20·8 km/sec ± 0·5 (p.e.) directed towards $l = 26°·1$, $b = + 16°·5$ (± 1°·2 p.e.). REDMAN finds from 392 K-type stars between 7·0 and 7·5 visual magnitude $V_0 = 20·5$ km/sec ± 1·3 (p.e.), $l = 31° ± 3°$ (p.e.), $b = + 17° ± 9°$ (p.e.)[1]). I have compared these values with the most recent results obtained by PLASKETT and PEARCE from 849 O—B7-stars brighter than $7^m·5$ and from 314 interstellar calcium velocities[2]). Taking the straight average of their solutions 1, 3 and 5 (Table 9; 1 and 3 being solutions without and with corrections for their "southern stream", 5 a solution from the interstellar gas) we obtain a solar velocity of 20·6 km/sec ± 0·4 (p.e.) pointing toward 25°·2 longitude ± 1°·4 (p.e.); the latitude of the apex cannot be determined with any certainty. The longitude and velocity agree with those derived by Mr VAN HOOF from the later type stars, and I adopt for the galactic regions the average between the two longitudes, and the latitude as derived by VAN HOOF, viz. $l = 25°·6$, $b = + 16°·5$, or $\alpha = 276°$, $\delta = + 30°$, while for the velocity the standard value of 20 km/sec has been adhered to. In higher latitudes the faint stars treated in the present paper must contain a rather larger percentage of high velocity stars as compared with the stars just considered. Estimates of the increase of the average velocity and of the relative number of high velocity objects have been made in *B.A.N.* No. 290 (c.f. Figure 1 and Table 7*a*). From these data and from the investigation of the effect of the high velocities upon the solar velocity as made by STRÖMBERG[3]) and by the author[4]) it was estimated that the solar velocity with respect to stars at 50° latitude will amount to about 24·2 km/sec for 11·9 photovisual magnitude and to about 22·8 km/sec for 11·0 photovisual magnitude[5]). The values are admittedly uncertain but I see no way for deriving much better results. For other latitudes the solar velocity was read from a smooth graph drawn through these points at 50° and the value 20·0 km/sec at 0°. With the aid of these values the mean parallaxes referred to above have been transformed into "secular parallaxes", the secular parallax υ_0 being defined as the motion of the sun in angular measure at the distance corresponding to the average parallax of the stars.

[1]) *Victoria Publications*, **6**, 37, 1931.
[2]) *Victoria Publications*, **5**, 274, Table 9, 1936.
[3]) *Astrophysical Journal*, **59**, 241 [formula (5)], 1924; *Mt. Wilson Contribution* No. 275.
[4]) *Groningen Publication* No. 40, Table 24, 1926.
[5]) In a recent article (*Astrophysical Journal*, **84**, 346, 1936) Miss WILLIAMS finds a *decrease* of solar velocity with distance from the galactic plane. However, the foundation for this result appears rather weak, and, moreover, it would apply only to regions very much closer to the galactic plane than those with which we are at present concerned.

For the lower latitudes, up to 30°, the values so obtained can be considered as sufficiently trustworthy; in higher latitudes the mean parallaxes are less accurate on account of lack of precise knowledge of the way in which the average peculiar velocity increases with distance from the galactic plane, while also the solar velocity is more uncertain. On the other hand the secular parallax derived from the systematic motions becomes relatively more reliable in this region because of the increased size of this parallax. For these reasons a compromise between the results derived from *B.A.N.* No. 290 [$\upsilon_0(\mu_i)$] and those of VAN DE KAMP and VYSSOTSKY [υ_0(McC)] has been adopted for this part of the sky. Table 2 shows a

TABLE 1.

Adopted values of the secular parallax υ_0 (unit ″/a·001) for photovisual magnitudes 11·0 and 11·9 (photographic 11·7 and 12·6).

b	$11^m·0$	$11^m·9$
0°	8·8	6·7
10	9·4	7·2
20	12·1	9·4
30	14·8	11·8
40	17·4	14·2
50	19·0	16·0
60	19·6	16·5
90	19·7	16·6

comparison between the two sets of results at 90° latitude. The McCormick values have been obtained by combining the four results of VAN DE KAMP and VYSSOTSKY's general solution for zones above ± 40° with the weights given[1]). They were increased by 2 % in order to reduce them to 90° latitude; the change of log υ_0 between 11·0 and 12·0 photovisual magnitude was assumed as ·06[2]). In the last column of Table 2 the corresponding results as computed by Mr VELDT[3]) with the aid of the luminosity law and star counts have been added for comparison.

TABLE 2.

Comparison of mean secular parallaxes at the galactic pole.

m_{pv}	$\upsilon_0(\mu_i)$	υ_0 (McC)	υ_0 (VELDT)
11·0	21·1	18·3	20·2
11·9	17·2	16·1	17·4

[1]) *Astronomical Journal* No. 1054, Table 2.2, or *Proceedings Nat. Ac. Washington*, **21**, 421, Table 1, 1935. The average photovisual magnitude to which these results refer is 11·2 according to the latter table.
[2]) This was inferred from the data of Table 3.1, l.c.
[3]) From Table 8, *B.A.N.* No. 290.

The straight average between $\nu_0(\mu_i)$ and ν_0(McC) has finally been adopted, which meant multiplying $\nu_0(\mu_i)$ by factors of ·934 and ·971 for $m_{pv} = 11{\cdot}0$ and 11·9 respectively; the same factors were applied for all latitudes between 40° and 90°, and smooth curves were drawn connecting the points so obtained and the original $\nu_0(\mu_i)$ for the lower latitudes with the latitude (Table 1).

The change of solar velocity on account of the increased percentage of high velocity stars will be accompanied by a slight shift in the apex. The shift corresponding to the change in solar velocity of 4·2 km/sec was estimated from Table 24, *Groningen Publication* No. 40, and the following positions of the apex were derived:

galactic latitude	0° to 20°	$\alpha = 276°$	$\delta = +31°$
	20 to 40	$\alpha = 279$	$\delta = +32$
	40 to 90	$\alpha = 282$	$\delta = +34$

Strictly speaking these apply to the faintest interval used, but they were adopted also for the stars between $10^{m}{\cdot}5$ and $11^{m}{\cdot}4$.

It should be remarked that the apex derived by VAN DE KAMP and VYSSOTSKY deviates somewhat from the apices I have adopted. Their general solution gave $\alpha = 19^{h}{\cdot}0 \pm 0^{h}{\cdot}2$ (p.e.), $\delta = +36° \pm 2°$ (p.e.)[1]), while a solution from comparison stars used in the parallax program of Boss stars at Allegheny and Johannesburg gave $\alpha = 18^{h}{\cdot}83 \pm 0^{h}{\cdot}14$ (p.e.); the average of the two deviates about 6° from the average of the positions adopted above. In view of the differences between the McCormick parallaxes north and south of the galactic plane such a difference in apex does not seem surprising; it may well have been caused by small systematic errors in the Boss motions.

Before using the secular parallaxes of Table 1 to correct the observed mean proper motions for the effect of the motion of the sun, I first multiplied them by a correcting factor F, by which I have tentatively allowed for the variation of the mean parallax with the star density within each zone of latitude. These factors were computed as follows. From Table 10, *Groningen Publication* No. 43, values of log $N(13{\cdot}0)$ were read corresponding to the galactic longitude of the group of regions considered and to the average galactic latitude of the latitude zone in which it occurs (+ 5°, − 5°, + 15°, − 15°, etc.); the photographic magnitude 13·0 was chosen as it corresponded approximately to the mean magnitude of the stars used. The average value of log $N(13{\cdot}0)$ for all areas in the same latitude zone, north and south combined, being denoted by $\overline{\log N(13{\cdot}0)}$, the correcting factors were found from the relation

$$\log F = -0{\cdot}4 \{\log N(13{\cdot}0) - \overline{\log N(13{\cdot}0)}\}.$$

[1]) L.c. p. 169.

The factor 0·4 has been roughly determined in an empirical way in *B.A.N.* No. 290. I have thus taken account of an eventual change of mean parallax with galactic longitude as well as with latitude.

In addition to the correction for the motion of the sun corrections were applied in order to eliminate the main effects of the rotation of the galactic system. If l and b are the galactic longitude and latitude[1]), l_0 the longitude of the galactic centre, A' and B' the well-known constants of differential rotation expressed in angular measure, μ_l and μ_b the projections of the total proper motion on the directions of increasing l and b respectively[2]), the corrections to be applied to the observed motions take the form:

$$\text{correction to } \mu_l = -\ A' \cos 2(l-l_0) \cos b - B' \cos b$$
$$\text{correction to } \mu_b = +\tfrac{1}{2} A' \sin 2(l-l_0) \sin 2b$$ [3]).

The values assumed for the various constants were as follows:

$$l_0 = 325°$$
$$A' = +\ ''/a{\cdot}0040$$
$$B' = -\ ''/a{\cdot}0027.$$

The longitude adopted for l_0 is in agreement with the average of the determinations tabulated by PLASKETT and PEARCE[4]) and with the galactic centre as derived from the distribution of globular clusters and planetary nebulae. The value of A' corresponds with $A = +{\cdot}019$ km/sec.parsec. This is slightly larger than the value adopted by PLASKETT and PEARCE[5]), $+{\cdot}0155 \pm {\cdot}0009$(p.e.). The true uncertainty is probably greater than that indicated by the probable error, as will be seen by comparing the four basic values in their Table 22[5]). The value referring to the fainter O—B2-stars, which received the largest weight, is uncertain on account of the small mean parallax of $''{\cdot}00094$, which can hardly be relied upon for stars of mean magnitude 6·74. I have preferred to use a mean parallax derived from that of the brighter group by assuming that these have the same mean absolute magnitude, and that there is an average absorption of $1^{m}{\cdot}5$ per 1000 parsecs. In this way I find $\bar{p} = ''{\cdot}00147$ and $A = {\cdot}020$ km/sec.parsec for the faint group. The last two of the four values referred to are less certain on account of the smallness of the

[1]) The position of the galactic pole used throughout the present article is that used in OHLSSON's Tables ($12^{h}40^{m}$, + 28°).

[2]) The notation μ_l as used in the present paper is not, therefore, equivalent with dl/dt.

[3]) VAN RHIJN has drawn my attention to the fact that this term has been erroneously omitted in *B.A.N.* No. 132. The omission of this small term cannot, however, have been of much consequence, the less so since the main discussion in that paper was confined to stars near the galactic plane, where the term vanishes.

[4]) *Victoria Publications*, **5**, 281, 1936.

[5]) L.c. pp. 293 and 294.

rotation effect for the later B-stars; giving these half weight compared with the O—B2-groups I find an average value of ·019. The only other independent results for A which have some weight are those derived from c-stars and bright δ Cephei variables [1]), yielding ·022 on the average (if we exclude the faintest c-star group on account of the preponderating influence of the Perseus region). I have finally adopted the value ·019, but should like to emphasize that the uncertainty of the constant is still rather great.

The value of B' is provisional; a correction to this constant is to be derived below from the McCormick motions.

If we denote the values of μ_l and μ_b remaining after the application of all these corrections by μ_l' and μ_b' the equations of condition take the following form:

$$\mu_l' = \Delta p' \{ \cdot 492 \cos b + \cdot 870 \sin (l - 153 \cdot 2) \sin b \} + \\ + \Delta\mu_\alpha' \{ \cdot 470 \cos b - \cdot 883 \sin l \sin b \} + \Delta B' \cos b$$

$$\mu_b' = \cdot 870\, \Delta p' \cos (l - 153 \cdot 2) - \cdot 883\, \Delta\mu_\alpha' \cos l.$$

In these formulae $\Delta p'$ represents the unknown additional [2]) correction to be applied to the constant of precession, $\Delta\mu_\alpha'$ is the constant error of the proper motions in right ascension remaining after the correction $\Delta p'$ has been applied, and due to the error of the zero point (in seconds of arc per annum) [3]), $\Delta B'$ is a correction to the adopted value of the constant B'.

A least-squares solution gave

m_{pv}	$\Delta p'$	p.e.	$\Delta\mu_\alpha'$	p.e.	$\Delta B'$	p.e.
	"/a		"/a		"/a	
11·0	+ ·0020		— ·0004		— ·0003	
11·9	+ ·0010		— ·0018		— ·0005	
combined	+ ·0015	± ·0013	— ·0011	± ·0013	— ·0004	± ·0005

The agreement between the values obtained for the two different magnitude intervals is very satisfactory. The results are practically identical with those derived by VAN DE KAMP and VYSSOTSKY from their solutions with 13 unknowns, which is somewhat unexpected in view of the considerable differences in the secular parallaxes which were used.

The probable errors were derived from the residuals of the various groups. The average residual is ± "/a·0042 for the groups at ± 5° and ± 15° latitude, and increases to a value about one and a half times larger for the zones at ± 50° and ± 70°. In the least-squares solutions the following weights were used:

Zone	Weight
± 5° and ± 15°	1·00
± 30°	·85
± 50°	·70
± 70°	·50

When undertaking the above solution I had hoped that the residuals might afford some insight into the systematic errors of the proper motions, but the accuracy does not yet seem sufficient to obtain useful results. In Tables 3*a* and 3*b* weighted means of $\mu_\alpha \cos \delta$ and μ_δ, both in thousandths of seconds of arc per annum, are shown arranged in wide intervals of right ascension and declination respectively. In the grouping with respect to right ascension regions with declinations higher than + 50° have been excluded.

TABLE 3*a*.
Residuals grouped according to right ascension.

α	$\mu_\alpha \cos \delta$	μ_δ	w
h h			
0·0 to 4·0	— 2·6	— 2·6	8
4·0 „ 8·0	+ ·8	— ·6	13
8·0 „ 12·0	+ 2·1	+ 2·3	8
12·0 „ 16·0	+ 1·6	— 3·1	7
16·0 „ 20·0	— 1·9	— 1·4	16
20·0 „ 24·0	— ·6	+ 3·0	14

TABLE 3*b*.
Residuals grouped according to declination.

δ	$\mu_\alpha \cos \delta$	μ_δ	w
— 25° to — 11°	— 1·1	+ ·8	13
— 10 „ + 4	— ·8	— ·2	12
+ 5 „ + 19	— 2·3	— ·5	13
+ 20 „ + 34	+ ·1	— 1·3	14
+ 35 „ + 49	+ 2·2	+ 1·1	13
+ 50 „ + 79	+ 4·1	— ·6	13

The weights are shown under w. The probable error of unit weight is ± 3·5, so that the average probable error of a tabulated value is ± 1·0.

The systematic errors of the corrected *Boss Catalogue* north of — 25° appear to be so small that a considerably greater accuracy of the relative proper motions would be necessary to disclose their character.

In Table 4 I have collected all the more important determinations from proper motions of the corrections to NEWCOMB's constant of precession, and of the zero point error of the motions in right ascension based on NEWCOMB's system, as well as the values found for the rotational constant B'. Three kinds of derivations may be distinguished, as indicated in the first

[1]) *B.A.N.* No. 132.
[2]) Provisional corrections $\Delta p = +$ "/a·0104 and $\Delta\mu_\alpha =$ — "/a·0110 to NEWCOMB's system had already been applied by VAN DE KAMP and VYSSOTSKY.
[3]) The correction to be applied to Boss's motions in right ascension is, therefore, — $\Delta\mu_\alpha/15$ in seconds of time. In previous articles the quantity $\Delta\mu_\alpha$ has usually been denoted as — Δe — $\Delta\lambda$.

TABLE 4.

Material	Reference	Δp	$\Delta\mu_\alpha$	p.e.	B'	p.e.
		"/a	"/a	"/a	"/a	"/a
(1) *Boss Catalogue*	BOSS-FOTHERINGHAM[1])	+ ·0093	— ·0114	± ·0020	— ·0017	± ·0006
(2) ,, ,,	OORT[2])	+ ·0100	— ·0120	± ·0013	— ·0023	± ·0005
(3) O-, B-, c-stars,	,,	+ ·0130	— ·0110	± ·0020	— ·0050	± ·0007
(4) etc.	PLASKETT and PEARCE[3])	+ ·0096	— ·0096	± ·0012	— ·0025	± ·0006
(5) Faint stars	VAN DE KAMP and VYSSOTSKY [4])	+ ·0101	— ·0107	± ·0010	— ·0030	± ·0006
(6) ,, ,,	,, ,,	+ ·0134	— ·0107	± ·0020	+ ·0047	± ·0016
(7) ,, ,,	OORT[5])	+ ·0119	— ·0121	± ·0013	— ·0031	± ·0005

column. First, those based on Boss's catalogue as a whole; second, those based on special distant stars situated between + 20° and — 20° galactic latitude; third, those based on faint stars directly connected up with Boss stars, as was done at the McCormick Observatory. Solution (6) was derived by VAN DE KAMP and VYSSOTSKY from the motions in right ascension of the comparison stars used in the determinations of parallax of Boss stars at the Allegheny Observatory and the Yale Southern Station. The restriction to μ_α makes the separation of Δp and $\Delta\mu_\alpha$ more difficult, and the determination of B' extremely poor. The first group of solutions has two disadvantages as compared with the others, viz. the much larger average peculiar motions, with the consequent necessity of excluding a number of large motions, and the danger that relatively slight systematic variations in the mean parallax (which averages "/a·070 approximately) with position in the sky (for instance with galactic longitude) would introduce spurious rotations. The determinations from B-stars and other distant objects have the disadvantage of being limited to galactic regions and influenced by local streams like that of the Scorpio-Centaurus cluster (PLASKETT and PEARCE's southern stream), the artificial removal of which would seem to present some danger[6]). Solutions (1) and (2) have been combined with equal weights, (3) and (4) similarly, (5), (6) and (7) with relative weights 10, 3 and 20 for Δp and $\Delta\mu_\alpha$, and with weights 10, $1\frac{1}{2}$ and 20 for B', the relative weights being estimated from the probability of each case. The average values for the three groups were then combined, with weights 1, 2 and 3 respectively, into the following means

$$\Delta p = + \text{"/a}{\cdot}0111 \pm \text{"/a}{\cdot}0010 \text{ (p.e.)}$$
$$\Delta\mu_\alpha = - \cdot 0111 \pm \cdot 0010 \text{ (p.e.)}$$
$$B' = - \cdot 0029 \pm \cdot 0005 \text{ (p.e.)}.$$

The value of B' may be compared with that which can be found from the constant A' by means of the theoretical relation

$$A/(-B) = (1/\alpha)^2 - 1,$$

where α is the ratio of the two axes of the velocity ellipsoid which are parallel to the galactic plane. With $\alpha = \cdot 63 \pm \cdot 02$ (p.e.)[1]) and $A' = +$"/a·0040 we obtain $B' = -$"/a·0026, in excellent agreement with the directly observed value.

If Δe is the correction required by Boss's motions on account of an error of NEWCOMB's motion of the equinox[2]), and $\Delta\lambda$ a correction to NEWCOMB's constant of planetary precession, we have

$$\Delta\mu_\alpha = -\Delta e - \Delta\lambda.$$

Extensive critical discussions of the correction to the motion of the equinox as indicated by modern observations have been given by KAHRSTEDT[3]) and MORGAN[4]). The former finds $\Delta e = -$"/a·0003; MORGAN, who includes recent Washington observations not used by KAHRSTEDT, obtains $\Delta e = -$ "/a·0010[5]). The probable error of the latter determination as

[1]) FOTHERINGHAM, *Monthly Notices R.A.S.* **86**, 419, 1926; Boss, Introduction to *Preliminary General Catalogue* p. XXIX.

[2]) *B.A.N.* No. 132, 1927.

[3]) *Publ. Victoria*, **5**, 294, 1936.

[4]) *Astronomical Journal*, **45**, 171, 1936 (Table 2.7). Solution (6) is from the proper motions in right ascension of the Allegheny-Yale comparison stars.

[5]) Present paper.

[6]) This southern stream must be largely responsible for the difference between the results in the third and fourth lines of the table. Compare also SMART, *Monthly Notices R.A.S.* **96**, 568, 1936.

[1]) *B.A.N.* No. 290, p. 83, 2nd column.

[2]) The notation is that of Boss's article on precession (*A.J.* No. 612, 1910). His definition of Δe as the correction required for NEWCOMB's value of the motion of the equinox, which I took over in *B.A.N.* No. 132, is somewhat ambiguous; in the sense in which it was used by Boss and the author it represented a correction to the motions based on NEWCOMB's equinox, and not to the equinox itself.

[3]) *Astronomische Nachrichten*, No. 5835, 1931.

[4]) *Astronomical Journal*, No. 969, 1931.

[5]) From occultations an independent value $\Delta e = +$ "/a·0054 ± "/a·0039 (p.e.) has been derived by SPENCER JONES in his revision of NEWCOMB's *Occultation Memoir* (*Monthly Notices R.A.S.* **90**, 89, 1929). A comparison of HORNSBY's observations of the sun between 1774 and 1798, published by KNOX-SHAW, JACKSON and ROBINSON in 1932, with modern observations gives $\Delta e = +$ "/a·009, or + "/a·005 if a correction of — s·025 is applied to the old observations (see above).

estimated from the internal agreement of the normal points used is roughly $\pm$ ″/a·0014, but the real uncertainty must be considerably greater, for the result depends to a great extent upon the amount of the systematic correction needed by the old eye-and-ear observations in order to reduce them to the chronograph method, and also upon the correction required to reduce the observations by fixed threads to moving threads. For example, the effect of the correction of $-$ s·025 applied by MORGAN to all old observations is equivalent with a change of approximately ″/a·0060 in Δe; the amount of the correction appears to be quite uncertain, and the resulting uncertainty of Δe must evidently be considerable. I have provisionally assumed a probable error of $\pm$ ″/a·0020 for Δe. For $\Delta\lambda$ a value $+$ ″/a·0003 $\pm$ ″/a·0010 has been derived by the late Professor DE SITTER in a still unpublished investigation on astronomical constants. Combining this with MORGAN's result for Δe we obtain $\Delta\mu_\alpha = +$ ″/a·0007 $\pm$ ″/a·0022, which deviates considerably from the value found from proper motions.

In order to derive the most probable values the direct determination of $\Delta\mu_\alpha$ should be combined by a least-squares solution with the two independent equations

$$\Delta p \cos \varepsilon + \Delta\mu_\alpha = - ''/a{\cdot}0010 \pm ''/a{\cdot}0005^{1)} \quad \text{(p.e.)}$$
$$\Delta p \sin \varepsilon = + \cdot 0044 \pm \cdot 0004 \quad ,,$$

which are equivalent with the corrections derived in this note. The question of the relative weights of the two determinations will remain to some extent a matter of personal taste, but there can be little divergence of opinion with regard to the fact that the determination of the equinox is subject to much greater difficulties than the measurement of differences in declination between stars at various right ascensions, upon which the proper motion results rest[2]). Adopting the probable errors as indicated I obtain as final corrections

$$\Delta p = + ''/a{\cdot}0089 \pm ''/a{\cdot}0010 \text{ (p.e.)}$$
$$\Delta\mu_\alpha = - \cdot 0088 \pm \cdot 0010 \quad ,, \quad (\Delta e = + ''/a{\cdot}0085)$$

The corresponding corrections to be applied to the proper motions reduced with the aid of NEWCOMB's constants, with their probable errors, are as follows:

$$\text{correction to } \mu_\alpha = + {}^{s}/a{\cdot}00004 - {}^{s}/a{\cdot}00024 \sin\alpha \operatorname{tg}\delta$$
$$\pm 3 \pm 3 \quad \text{(p.e.)}$$
$$\text{correction to } \mu_\delta = -''/a{\cdot}0035 \cos\alpha.$$
$$\pm 4 \quad ,,$$

1) The probable error of the quantity $\Delta p \cos \varepsilon + \Delta\mu_\alpha$ representing the total average of all proper motions in right ascension is of the same order as that of B'. The greater uncertainty of Δp and $\Delta\mu_\alpha$ is, of course, due to the difficulty of separating these two unknowns.

2) A combined solution of this type has been given by LEWIS BOSS in the introduction to his *Preliminary General Catalogue*, p. XXIX (compare also *Astronomical Journal*, **26**, 95, 1910). Boss adopts a ratio of 10 : 1 for the weights of the quantity $\Delta p \cos \varepsilon + \Delta\mu_\alpha$ derived from μ_α, and the direct determination of Δe, while the ratio assumed above is as 16 : 1.