Investigations on Population II Cepheids.IV. Colours and luminosities

Kwee, K.K.

1968, *Bull. Astr. Inst. Netherlands* **19** 374–390; *Communication from the Observatory at Leiden*

INVESTIGATIONS ON POPULATION II CEPHEIDS
IV. COLOURS AND LUMINOSITIES

K. K. KWEE

Received 9 November 1967

In this fourth and last part of a series of publications on Population II Cepheids, some relations between luminosity, amplitude, colour, period and spectral type are investigated. In the *B* amplitude-log *P* diagram, five out of six galactic Population II Cepheids with periods of about two days exceed the upper limit defined by Kraft for classical Cepheids. As compared with the colours, the spectral types found by Wallerstein are usually half a spectral class too early. Abt's spectral determinations for W Vir are, however, in fair agreement with the *B*−*V* colours determined here. A comparison with the Cepheids found in globular clusters reveals that the distinction of three major groups of Population II Cepheids, viz. short-period variables with large amplitudes, crested and flat-topped Cepheids, is also present among these. The meagre available data indicate that the first-mentioned group has a spectral range of about A5–F5 and luminosity class II or III, the crested variables have a range of A8–G0 Ib and the flat-topped ones F2–G5 Ib. The plots of absolute luminosity and intrinsic colour against log *P* for the globular-cluster Cepheids show different relations for the variables with periods shorter than three days and those with periods longer than three days. There are also indications that the crested variables are about half a magnitude brighter than the flat-topped Population II Cepheids. It is remarkable that crested variables only occur in globular clusters with an intermediate metal content and the flat-topped Cepheids in metal-poor clusters. Finally, colour excesses, intrinsic colours and distances have been determined for the 23 galactic Population II Cepheids discussed in this investigation.

1. Introduction

This is the fourth and last publication of a series of papers on Population II Cepheids. In the first paper of this series (KWEE and BRAUN, 1967) new photo-electric *UBV* observations have been given for 32 pulsating variables, some two dozen of which are galactic Population II Cepheids. The second paper (KWEE, 1967a) dealt with a discussion of the periods and their variations. In the third paper (KWEE, 1967b) the light-curves and two-colour diagrams have been investigated. It was found that for those Population II Cepheids with periods between 13 and 20 days, two types of light-curves can be distinguished, the crested and the flat-topped light-curves. The two-colour diagrams show generally more open loops for the Population II Cepheids than for the classical Cepheids. In the present paper relations between luminosity, amplitude, colour, period and spectral type of the investigated Population II Cepheids are discussed, and a comparison is made with the Population II Cepheids found in globular clusters and in the Andromeda nebula. Also an attempt is made to derive distances for the variables.

2. The magnitudes and colours at maximum and minimum phases, and the amplitudes

In table 1 the extreme magnitudes and the corresponding colours of the investigated Cepheids have been given. The variables have been arranged according to period and a separation has been made between the 24 Population II Cepheids and the five classical Cepheids (last entries) observed in the present investigation. In the first column the name of the variable has been listed, columns 2 and 3 give the approximate period and its logarithm, respectively, and columns 4 to 9 the *V* magnitudes and the *B*−*V* and *U*−*B* colours at maximum and at minimum phases. It should be stressed that the tabulated colours are not the extreme colours possibly attained by the variable, because for the majority of the listed variables the latter colours actually occur at some different phases. Finally, columns 10 to 14 give the amplitudes in *V*, *B* and *U*, and the actual range in the variations of the colours *B*−*V* and *U*−*B*, respectively. All the results in the table were derived by using only the observations discussed in the first paper of this series (KWEE and BRAUN, 1967).

TABLE 1

Magnitudes and colours at maximum and minimum phases, and amplitudes of the investigated Cepheids

Name of variable	P (days)	log P	Maximum			Minimum			Amplitude				
			V	B−V (mag)	U−B	V	B−V (mag)	U−B	V	B	U (mag)	B−V	U−B
V 527 Sgr	1.26	0.100	(14.37)	(0.31)	(−0.04)	15.38	0.60	0.22	(1.01)	(1.30)	(1.56)	(0.33)	(0.26)
VZ Aql	1.67	0.223	13.21	0.59	0.39	(14.04)	1.03	0.65	(0.83)	(1.27)	(1.53)	0.47	0.29
V 839 Sgr	1.84	0.265	14.52	0.47	0.17	15.31	0.73	0.43	0.79	1.05	1.31	0.39	0.43
TU Cas	2.14	0.330	var.	var.	var.	var.	var.	var.	var.	var.	var.	var.	0.17
UY Eri	2.21	0.344	11.03	0.36	0.11	11.64	0.51	0.08	0.61	0.76	0.73	0.23	−0.06
AU Peg	2.40	0.380	9.03	0.75	0.53	9.44	0.93	0.65	0.41	0.59	0.71	0.21	0.20
V 465 Oph	2.84	0.453	12.99	0.66	0.51	13.98	1.09	0.68	0.99	1.42	1.59	0.46	0.32
AP Her	10.4	1.017	10.36	0.65	0.44	11.11	0.96	0.59	0.75	1.06	1.21	0.35	0.25
V 1077 Sgr	13.4	1.127	(12.45)	0.55	0.24	13.74	0.80	0.42	(1.29)	(1.54)	(1.72)	0.42	(0.44)
V 802 Sgr	13.5	1.130	13.35	0.96	0.50	14.31	1.30	1.26	0.96	1.30	2.06	0.48	(0.90)
V 410 Sgr	13.8	1.140	12.15	0.74	0.42	13.12	1.10	0.80	0.97	1.33	1.71	0.50	0.63
CS Cas	14.7	1.167	11.46	0.58	0.36	(12.90)	(1.12)	(0.65)	(1.44)	(1.98)	(2.27)	(0.63)	0.71
FI Sct	14.9	1.173	13.52	1.38	(0.83)	14.71	1.65	(1.20)	1.19	1.46	(1.83)	(0.40)	(0.43)
V 1187 Sgr	15.1	1.179	13.47	(0.88)	(0.40)	14.65	(1.16)	(0.87)	1.18	(1.46)	(1.93)	(0.67)	(0.67)
V 741 Sgr	15.2	1.182	12.38	0.80	0.43	13.36	1.20	0.94	0.98	1.38	1.89	(0.40)	(0.54)
CZ Sct	15.4	1.188	14.05	1.08	0.79	15.10	1.65	(1.23)	1.05	1.62	(2.06)	0.61	(0.65)
AL Sct	15.6	1.193	13.60	1.06	0.70	14.88	1.56	(1.48)	1.28	1.78	(2.56)	0.53	(0.85)
V 377 Sgr	16.2	1.210	12.96	0.93	0.52	13.92	1.21	0.84	0.96	1.24	1.56	(0.32)	0.60
V 478 Oph	16.4	1.215	12.56	0.97	0.48	13.49	1.31	0.80	0.93	1.27	1.59	(0.42)	(0.56)
CO Sct	17.1	1.233	13.64	1.27	1.00	15.09	1.68	(1.38)	1.45	1.86	(2.24)	0.77	(0.80)
W Vir	17.3	1.238	9.55	0.49	0.17	(10.72)	(0.91)	(0.60)	(1.18)	(1.60)	(2.03)	0.57	0.60
V 1303 Sgr	18.5	1.267	12.36	0.68	0.40	13.59	1.10	0.53	1.24	1.66	1.79	0.57	0.52
CC Lyr	24.0	1.380	11.65	0.31	0.16	12.28	0.64	0.19	0.63	0.96	0.99	0.34	0.19
TW Cap	28.6	1.456	(10.00)	(0.33)	(0.18)	11.13	0.70	0.28	(1.13)	(1.50)	(1.60)	(0.44)	0.30
V 532 Cyg	3.28	0.516	8.88	1.02	0.69	9.24	1.19	0.78	0.36	0.53	0.62	0.17	0.11
BB Her	7.51	0.876	9.79	0.90	0.64	10.42	1.25	0.99	0.63	0.98	1.33	0.35	0.39
VY Cyg	7.86	0.895	9.18	1.08	0.78	10.09	1.45	1.10	0.91	1.28	1.60	0.40	0.41
SZ Cyg	15.1	1.179	8.94	1.28	0.98	(9.86)	(1.80)	(1.65)	(0.92)	(1.44)	(2.11)	0.59	0.82
CD Cyg	17.1	1.233	8.38	0.97	0.75	9.54	1.63	1.49	1.16	1.82	2.56	0.67	0.83

Parentheses indicate that the phases in question were not directly observed and, therefore, the corresponding magnitudes and colours were estimated from extrapolations based on the shape of the light-curve found by others. For the very faint variables (viz. those in Scutum) the $U-B$ colours, in particular at minimum, are uncertain. In the case of UY Eri the minus sign in the $U-B$ amplitude means that the minimum of the $U-B$ colour-curve occurs when the variable is at its maximum and vice versa. This appears to be a common feature for RR Lyrae variables, where the shape of the $U-B$ colour-curve is roughly a mirror image of that of the integrated light-curve. This reversal in the $U-B$ curve of UY Eri has led to the negative slope in the two-colour diagram for this variable, mentioned in the preceding paper of this series (KWEE, 1967b), and indicates at least a different type of variability for this star than for the other short-period Population II

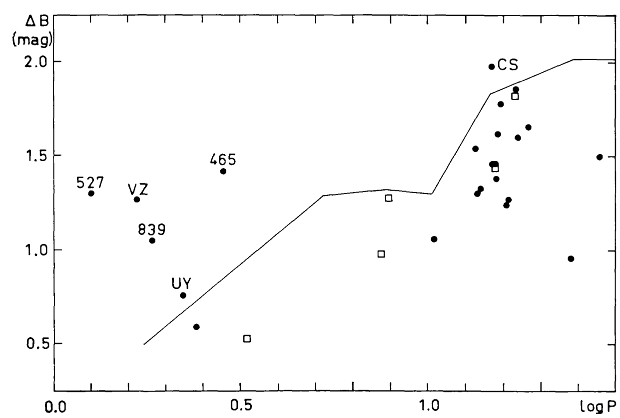

Figure 1. Amplitude-period diagram for the investigated Population II Cepheids. In this figure the B amplitudes of the variables from table 1 are plotted against the logarithm of their periods. The variable TU Cas has been omitted. The broken line indicates Kraft's upper limit for classical Cepheids. Dots represent the Population II and squares the classical Cepheids of the present investigation. The six variables falling above Kraft's limit are marked by their variable-star letters or numbers.

Cepheids. Even the possibility that this variable is an RR Lyrae variable with a much shorter period is not excluded.

In figure 1 the B amplitude-log P diagram has been given for all the Cepheids of this investigation except TU Cas, because this variable has a variable amplitude. In such a diagram the amplitudes of the classical Cepheids have an upper limit which depends upon the period (cf. OOSTERHOFF, 1960; KRAFT, 1961). Kraft's upper limit has been drawn as the broken line in the diagram. Different symbols have been used for the classical Cepheids and the Population II Cepheids. It appears that six of the present Population II Cepheids exceed the upper limit found by Kraft. In the figure these six variables have been indicated by their variable-star numbers or letters. Among them are five variables with periods around two days: VZ Aql, V 465 Oph, V 527 Sgr, V 839 Sgr and UY Eri. The first four, for which the discrepancy is largest, all show light- and colour-curves similar in shape (cf. KWEE, 1967b).

Figure 2 shows a plot of the amplitude in $U-B$ against that in $B-V$. In this diagram TU Cas and UY Eri have been omitted. The first because of its variable amplitude and the second because of the negative $U-B$ amplitude mentioned above. The straight line in the figure represents the relation derived by OOSTERHOFF (1960) for classical Cepheids. Three conclusions can be drawn from this diagram. First, the relation derived by Oosterhoff does not represent the classical Cepheids with large amplitudes. This can also be seen from Oosterhoff's original diagram. Secondly, there is no difference between the Population II Cepheids and the classical Cepheids of the present investigation. And finally, there is a clear distinction between the Population II Cepheids with periods between 13 and 20 days and the other Population II Cepheids. The former appear to have the larger amplitudes in the colours.

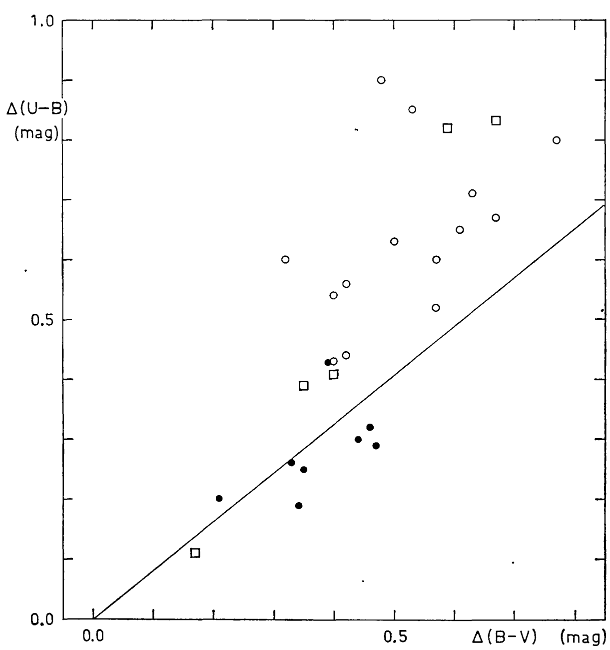

Figure 2. Plot of the amplitude in $U-B$ against the amplitude in $B-V$. All the investigated Cepheids are included in this figure, except TU Cas and UY Eri. Squares represent the classical Cepheids, open circles the Population II Cepheids with periods between 13 and 20 days, and dots correspond with the other Population II Cepheids. The straight line represents Oosterhoff's relation.

In the preceding paper of this series (KWEE, 1967b) a distinction was made between crested and flat-topped light-curves of Population II Cepheids with periods between 13 and 20 days. It is interesting to investigate whether this division can be traced in the amplitudes of the variables of the different groups. From table 1 and figures 1 and 2 it can be seen that the dispersion in the amplitudes for all the colours is relatively large for this range of periods. Therefore in table 2 average values have been given for the two groups of variables mentioned. For comparison the corresponding values for the four short-period Cepheids, VZ Aql, V 465 Oph, V 527 Sgr and V 839 Sgr, have also been given in the table. The numbers between parentheses indicate the corresponding standard deviations within each group. It can be concluded that, although no significant differences exist between the amplitudes of the crested and those of the flat-topped variables, the amplitudes in V and B of the first group seem to be systematically larger by a few tenths of a magnitude. The four short-period Cepheids mentioned in table 2 show, however, amplitudes which are in general about 25 per cent smaller than those of the variables with periods between 13 and 20 days. In particular in $U-B$ the amplitude is relatively very small.

3. Comparison with the Cepheids found in globular clusters

According to the *Second Catalogue of Variable Stars in Globular Clusters* (SAWYER, 1955), there are 30 Cepheids known in globular clusters. For 26 of them

TABLE 2

Comparison between the amplitudes of the three major groups of Population II Cepheids

Group	Crested		Flat-topped		Short-period	
	V 1077 Sgr V 410 Sgr CS Cas FI Sct CO Sct V 1303 Sgr		V 802 Sgr V 1187 Sgr V 741 Sgr CZ Sct AL Sct V 377 Sgr V 478 Oph W Vir		V 527 Sgr VZ Aql V 839 Sgr V 465 Oph	
Range in period	$13^d.4$–$18^d.5$		$13^d.5$–$17^d.3$		$1^d.26$–$2^d.84$	
Amplitude in V	1.26	(0.18)	1.06	(0.13)	0.90	(0.11)
B	1.64	(0.25)	1.46	(0.19)	1.26	(0.15)
U	1.93	(0.26)	1.96	(0.31)	1.50	(0.13)
$B-V$	0.55	(0.14)	0.50	(0.12)	0.41	(0.07)
$U-B$	0.59	(0.15)	0.67	(0.14)	0.32	(0.07)

periods have been established. Ten variables have periods between 1 and 3 days, three between 4 and 8 days, eleven between 13 and 20 days, and two have periods of about 26 days. Photo-electric *UBV* photometry on six of these variables has been carried out by ARP (1957) and by EGGEN (1961). Arp observed the variables M 10 No. 3 ($P = 7^d.908$), M 10 No. 2 ($P = 18^d.728$), M 5 No. 42 ($P = 25^d.728$) and M 5 No. 84 ($P = 16^d.62$), while Eggen made observations of ω Cen No. 92 ($P = 1^d.345$) and M 80 No. 1 ($P = 15^d.70$).

Very striking is the similarity in period, light- and colour-curves of M 5 No. 42 and the galactic Population II Cepheid TW Cap. The same can be said of M 5 No. 84 and TW Cap, except for the cycle-to-cycle variations in the light-curve of the former. Plots of the two-colour diagrams of these two M 5 variables are shown in figure 3 (a and b). The loops drawn in this figure correspond with the track for TW Cap (KWEE, 1967b), assuming a difference in interstellar reddening in $B-V$ of 0.08 mag between TW Cap and the M 5 variables. According to FERNIE (1964) there is no colour excess for M 5, so that it may be concluded that TW Cap has a colour excess $E_{B-V} = 0.08$ mag, due to interstellar absorption. WALLERSTEIN (1958) gave the same spectral types (within two subdivisions) for all three variables.

ARP (1957) has pointed out that the period and the light- and colour-curves of the globular-cluster Cepheid M 10 No. 2 are also very similar to those of the galactic Population II Cepheid W Vir. According to WALLERSTEIN (1958) the spectral types of the two stars are also quite similar. Although Arp did not publish a list of his individual observations of the M 10 variable, it was possible to construct a two-colour diagram by reading off the $B-V$ and $U-B$ observations from figure 3 of his paper (ARP, 1957). The result is plotted in figure 3 (c) of the present paper. It can be seen from this figure that the plots can be made fairly well coincident with the loop of W Vir (KWEE, 1967b) when a difference in interstellar reddening in $B-V$ of about 0.18 mag is assumed. For the phases near maximum the agreement is much better than for the four observations near minimum, where Arp already found an abnormal ultraviolet excess. FERNIE (1964) derived a colour excess $E_{B-V} = 0.24$ mag for the cluster M 10. This leaves an excess of about 0.06 mag for the galactic Population II Cepheid W Vir.

The variable M 80 No. 1 is probably also a flat-topped variable like W Vir and M 10 No. 2. EGGEN's (1961) observations, however, show that the amplitudes in V, $B-V$ as well as in $U-B$ are about 40 per cent smaller than those of the two variables mentioned. The two-colour diagram for M 80 No. 1 has been plotted in figure 3 (d). Eggen found a reddening of at least $E_{B-V} = 0.20$ mag for the cluster. Adopting a reddening $E_{B-V} = 0.06$ mag for W Vir as derived above, a loop for W Vir has been drawn in the figure which was artificially reddened in $B-V$ by 0.14 mag. The figure shows that the plots of M 80 No. 1 are systematically higher than the adjusted loop for W Vir. This should probably be explained as a $U-B$ excess of about

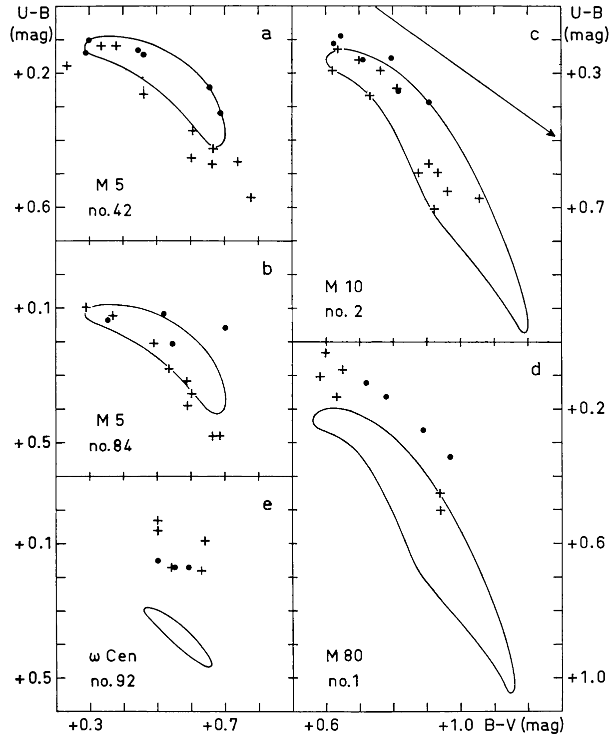

Figure 3. Two-colour diagrams for five globular-cluster Cepheids, from photo-electric observations made by Arp (M 5 and M 10) and by Eggen (M 80 and ω Cen). Dots correspond to observations on the rising branch of the $U-B$ curve and crosses to those on the descending branch. The arrow in the upper right corner of the figure indicates the direction of the interstellar reddening. The loops drawn in each plot are those of the following galactic Population II Cepheids: a) and b) TW Cap corrected for an interstellar colour excess E_{B-V} of 0.08 mag; c) W Vir artificially reddened with an extra interstellar colour excess E_{B-V} of 0.18 mag; d) W Vir artificially reddened with an extra interstellar colour excess E_{B-V} of 0.14 mag; and e) AU Peg corrected for an interstellar colour excess E_{B-V} of 0.22 mag.

0.15 mag compared with W Vir.

The variable M 10 No. 3 is the only Cepheid with a period of around seven days found in a globular cluster. ARP's (1957) photo-electric *UBV* observations show that the light- and colour-curves of this variable are quite unlike those of the galactic Cepheids VY Cyg and BB Her observed in this programme. This fact has strengthened the present conclusion that BB Her is a classical Cepheid like VY Cyg, and not a Population II Cepheid, as KUKARKIN *et al.* (1958, 1960) have classified it.

The globular-cluster Cepheid ω Cen No. 92 (EGGEN, 1961) and the galactic Population II Cepheid AU Peg have one feature in their light-curves in common. Both have amplitudes of only about 0.4 mag in V, which is half of the corresponding amplitudes of the majority of Population II Cepheids with periods of around two days. However, their periods differ by nearly a factor two. Moreover, their positions in the two-colour diagram also differ considerably from each other as is shown in figure 3 (e). In this figure the loop for AU Peg (KWEE, 1967b) has been corrected for a differential interstellar reddening E_{B-V} of 0.22 mag. The adoption of this amount of differential reddening is based on equal intrinsic $B-V$ colours for the two stars. It can be seen from the figure that in this case ω Cen No. 92 has an excess in $U-B$ of about 0.25 mag compared with AU Peg.

Besides the three-colour photo-electric investigations mentioned above, ARP (1955) and DICKENS and CAREY (1967) made two-colour photographic observations of 13 other Cepheids in globular clusters. Dickens and Carey investigated the variables in ω Cen, while Arp did the same with variables in other globular clusters. Reviewing this photographic material, it is found that, except for the variable ω Cen No. 92 discussed above, there are six other globular-cluster variables with periods between one and three days. These are ω Cen Nos. 43, 60 and 61, M 13 Nos. 1 and 6, and M 15 No. 1. All six show rather complicated light-curves with large amplitudes. In this respect they closely resemble the four galactic Population II Cepheids of the present investigation: VZ Aql, V 465 Oph, V 527 Sgr and V 839 Sgr.

Six globular-cluster variables included in the photographic investigations have periods between 13 and 20 days. Apart for M 10 No. 2, which has been discussed above, the variable ω Cen No. 29 and the M 2 variables Nos. 1, 5 and 6 apparently belong to the group of flat-topped variables introduced in the preceding paper of this series (KWEE, 1967b). Only one variable, M 3 No. 154, is a crested variable.

In table 3 a summary has been given comparing the galactic Population II Cepheids of the present investigation with those Cepheids of the globular clusters which have been observed in two or three colours. The variables are all grouped according to period. In column 3 the galactic variables are listed, while columns 4 and 5 give the Cepheids of the globular clusters which have been investigated in three and in two colours, respectively.

TABLE 3
Corresponding groups of Population II Cepheids

Group	Period (days)	Galactic Cepheids	Cepheids in globular clusters	
			three-colour obs.	two-colour obs.
Short-period Cepheids with large amplitudes	1–3	V 527 Sgr VZ Aql V 839 Sgr V 465 Oph		ω Cen No. 43 ω Cen No. 60 M 15 No. 1 M 13 No. 1 M 13 No. 6 ω Cen No. 61
Short-period Cepheids with small amplitudes	1–3	UY Eri? AU Peg	ω Cen No. 92	ω Cen No. 92
	4.47 5.11			ω Cen No. 48 M 13 No. 2
	7.87		M 10 No. 3	M 10 No. 3
	10.4	AP Her		
Crested Cepheids	13–20	V 1077 Sgr V 410 Sgr CS Cas FI Sct CO Sct V 1303 Sgr		M 3 No. 154
Flat-topped Cepheids	13–20	V 802 Sgr V 1187 Sgr V 741 Sgr CZ Sct AL Sct V 377 Sgr V 478 Oph W Vir	M 80 No. 1 M 10 No. 2	ω Cen No. 29 M 2 No. 1 M 2 No. 5 M 10 No. 2 M 2 No. 6
	24.0	CC Lyr		
	25.7 26.6 28.6	 TW Cap	M 5 No. 42 M 5 No. 84	M 5 No. 42 M 5 No. 84

It should be emphasized that, by the present material, only the distinction of the short-period Cepheids with large amplitudes, the crested Cepheids and the flat-topped variables is well established. Concerning the others, the grouping as made in the table is very tentative, because of the small samples involved. As for the short-period Cepheids with small amplitudes, for example, it is doubtful whether AU Peg and ω Cen No. 92 are physically alike, since it has been shown above that their intrinsic $U-B$ colours differ so much. The variables ω Cen No. 48 and M 13 No. 2, with periods of around five days, might form an extension to larger periods of the group of short-period Cepheids with large amplitudes. The amplitudes of these two variables are of the same order as those of the other group. As far as the light-curve is concerned, M 10 No. 3, with a period of $7^d.87$, might also belong to the same group. It is likely, however, judging from the period, light-curve and colours, that the variable AP Her is not related to any of the three groups mentioned above. Finally, the four variables in the table with periods longer than 20 days are probably related to the crested variables. The resemblance between TW Cap and the two M 5 variables has already been mentioned. The light-curves of TW Cap and CC Lyr show the same characteristics as that of a crested variable, except that for CC Lyr the amplitude is much smaller and the light-curve is probably variable (KWEE, 1967b).

4. Comparison between the observed colours and the spectral types

Up to the present time spectral investigations on galactic Population II Cepheids have been neglected very much. This is probably due to the faintness of most of these objects. Only for the two brightest Population II Cepheids, W Vir and TW Cap, a more or less detailed study has been made (ABT, 1954; WALLERSTEIN, 1958).

For W Vir, Abt and Wallerstein both agreed on a luminosity class of Ib. With respect to the spectral type, however, they differ by about half a spectral class. Wallerstein found spectral classes ranging from F0 at light-maximum to G0 at light-minimum. Abt, on the other hand, indicated a range from F2 (just before maximum) to G6 (at minimum). In the case of TW Cap

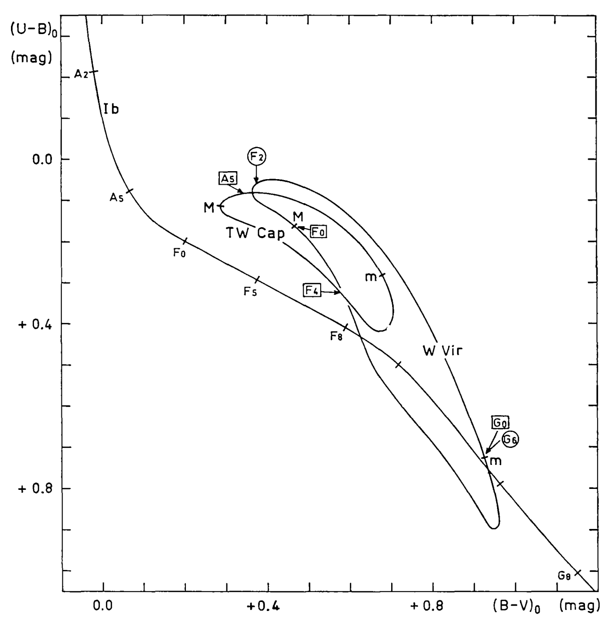

Figure 4. Comparison of spectral types and colours for TW Cap and W Vir. In this two-colour diagram the loops of TW Cap and W Vir are drawn after corrections have been applied for interstellar reddening by the amounts of E_{B-V} = 0.08 and 0.06 mag, respectively. The curve drawn in the diagram represents the supergiant series of luminosity class Ib. The sequence of spectral types, according to Schmidt-Kaler, has been indicated along this track. The loops of the two variables are marked with the spectral classifications made by Wallerstein (enclosed by a square) and by Abt (encircled) at the various phases. Maximum and minimum phases are indicated by M and m, respectively.

Wallerstein found spectral types ranging from A5 (just before maximum) to F4 (.4 later in phase). In figure 4 the loops of both stars in the two-colour diagram are shown schematically. The colours have been corrected for the small amounts of interstellar reddening mentioned in section 3 and corresponding with E_{B-V} = 0.06 and 0.08 mag for W Vir and TW Cap, respectively. Also indicated in the figure is the track for the Ib supergiants, marked with the sequence of spectral classes according to SCHMIDT-KALER (1965). Furthermore, the spectral classifications of Abt and Wallerstein have been indicated in the figure at the positions corresponding to the proper phases of the light-variation.

Apart from the fact that the loops generally lie above the line of the supergiants, there are three remarkable features to be noted. First, the difference in spectral type found by Wallerstein between W Vir and TW Cap corresponds with the difference in the $(B-V)_0$ colour only. These colours are bluer for the latter star, while the $(U-B)_0$ colours at maximum are about the same. Secondly, as already mentioned, the large discrepancies between Abt's and Wallerstein's spectral determinations for W Vir are conspicuous, although both used metallic lines for their classifications. And thirdly, Abt's classification for W Vir is in agreement with the $(B-V)_0$ colours, while Wallerstein's determinations for W Vir as well as for TW Cap do not agree with the $(B-V)_0$ nor with the $(U-B)_0$ colours, but are in fact much too early with respect to these colours.

Two other variables of the present investigation have been classified spectroscopically by WALLERSTEIN (1958). These are AP Her ($P = 10^d.4$) and UY Eri ($P = 2^d.21$). In his table 2 Wallerstein gave for AP Her a spectral range of F0–F5, with the remark that it might even be greater. The photo-electric observations show that, in order to make the $B-V$ colour of the variable at maximum coincident with that of an F0 Ib supergiant, a colour excess in $B-V$ of 0.45 mag due to interstellar reddening should be assumed. In figure 5 a two-colour diagram is given with the observed loop of AP Her (KWEE, 1967b) and the loop corrected for an interstellar reddening of this amount. The curves for Ib supergiants and normal giants of luminosity class III, marked with the sequences of spectral types (SCHMIDT-KALER, 1965), are also drawn in the figure. It can be seen from this that the $B-V$ colours indicate a spectral range of F0–F8, which is not discordant with Waller-

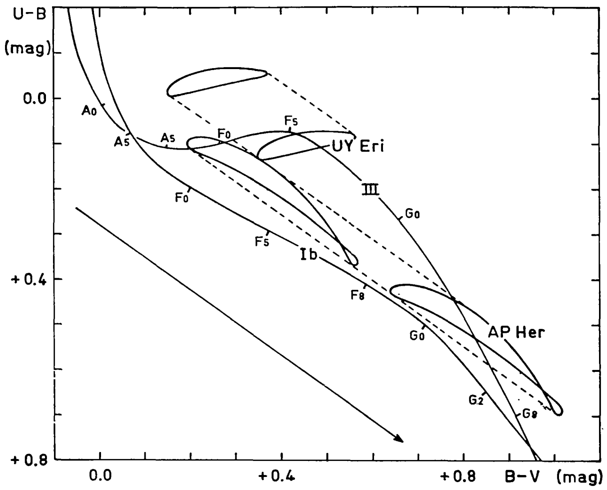

Figure 5. Comparison of spectral types and colours for AP Her and UY Eri. In this two-colour diagram schematic loops are shown for AP Her and UY Eri. The lower right loop of each pair corresponds to the uncorrected position, while the upper left loop indicates the true position if hypothetical reddening shifts of $E_{B-V} = 0.45$ and 0.20 mag are assumed for the two stars, respectively. Also drawn in the figure are the tracks for the supergiants of luminosity class Ib and the giants of luminosity class III after Schmidt-Kaler. Both tracks are marked with spectral sequences. The arrow indicates the direction of increasing interstellar reddening.

stein's remark. Furthermore, compared with the Ib-supergiant series, a slight excess in $U-B$ of 0.1 mag is present.

In the case of UY Eri, WALLERSTEIN (1958) gave a spectral range of A5–F2. In figure 5 the observed loop for this variable has also been drawn schematically with a negative slope as found in section 2. Because of this and also because of the large deviation of the observed loop above the Ib-supergiant track, it is probably more useful to compare UY Eri with the normal giants of luminosity class III instead of with the supergiants. Assuming the intrinsic $B-V$ colours of the variable to be the same as those of A5 to F2 giants of luminosity class III, it is found that the excess due to interstellar reddening in this colour is about 0.20 mag. From figure 5 it can be seen that in this case an excess in $U-B$ of about 0.15 mag is present. However, a reddening of this amount for UY Eri should be considered as very extreme because the galactic latitude of this variable is $-53°$. It is, therefore, more likely that the colour excess due to interstellar reddening is much smaller than this amount. This means that Wallerstein's spectral types are too early for the observed colours.

For three other Population II Cepheids in this investigation, AU Peg ($P = 2^d.40$), V 377 Sgr ($P = 16^d.2$) and V 410 Sgr ($P = 13^d.8$), spectral types have been given by KUKARKIN *et al.* (1958, 1960). However, the information is insufficient to make a comparison with the observed colours useful.

It is very tempting to look for spectral differences between the members of the three groups of Population II Cepheids: the short-period, crested and flat-topped variables. However, for the Cepheids in the galactic field such an investigation will at present necessarily be incomplete because of the lack of sufficient spectral information. In particular, no spectral investigation has yet been made of any of the crested galactic variables listed in table 3. The Cepheid TW Cap, which indeed has a crested light-curve, shows spectral types which are earlier than those of the flat-topped variable W Vir (WALLERSTEIN, 1958). However, it is hard to say to what extent this is due to the difference in period between the two variables. Of the short-period variables in the galactic field listed in table 3 spectral information is available only for UY Eri, but this variable is different in behaviour from the other Population II Cepheids in this period range and therefore it cannot be considered a representative of this group. There is, however, another galactic short-period Population II Cepheid which can serve for the present purpose. This variable is BL Her ($P = 1^d.31$), which has been investigated photo-electrically and spectroscopically by ABT and HARDIE (1960). According to the light- and colour-curves this star is very similar to VZ Aql, V 465 Oph, V 527 Sgr and V 839 Sgr. Abt and Hardie gave a range in spectral type for this variable of F0–F6 and furthermore assigned a luminosity class of II or III to the variable. WALLERSTEIN (1958) also examined this star spectroscopically and arrived at a spectral range of A6–F3. Here again Wallerstein's spectral classifications are about half a spectral type earlier than those made by Abt. Assuming BL Her to be a representative short-period Population II Cepheid with a large amplitude and W Vir a representative flat-topped variable, Abt's and Wallerstein's results both indicate earlier spectral types for the first-mentioned group than for the second one. Furthermore, the luminosity classes are also probably different, in the sense that the short-period variables are less luminous (class II or III) than the Population II Cepheids with longer periods (class Ib).

Further information on the spectroscopic properties of the Population II Cepheids can be found in spectral investigations of the globular-cluster Cepheids. JOY (1949) investigated a great number of these variables spectroscopically, among which several of those listed in table 3, viz. three of the short-period variables with large amplitudes, M 13 Nos. 1 and 6 and M 15 No. 1, the crested variable M 3 No. 154, and five of the flat-topped variables, M 2 Nos. 1, 5 and 6, M 10 No. 2 and M 80 No. 1. His results revealed no appreciable differences between the spectral types of the crested and the flat-topped variables. Both categories appeared to have a range from F5 to G2 or G3. The short-period variables, however, had a spectral range of A2–F5, which was definitely earlier than for the longer periods.

WALLERSTEIN (1958) also made an examination of spectra of some globular-cluster Cepheids, among which are M 13 No. 1 (short-period), M 3 No. 154 (crested) and M 10 No. 2 (flat-topped). Although he also made use of some of Joy's original spectra, Wallerstein's classifications are generally somewhat earlier than Joy's spectral types. Differentially, however, he arrived at the same conclusion that the spectral types for the short-period variables are earlier than for the Cepheids with longer periods. Contrary to Joy he also found that at its earliest the spectrum of a crested variable is about half a spectral class earlier than the corresponding spectrum of a flat-topped variable.

Summarizing all these spectroscopic data, the following conclusions can be made. The short-period Population II Cepheids with large amplitudes have a spectral range of about A5–F5 and a luminosity class II or III. The flat-topped Population II Cepheids have spectral types which are about one class later than these; they range from about F2 to G5 and are usually of luminosity class Ib. The crested variables are probably earlier in spectral type by about half a spectral class than the flat-topped Cepheids, but have the same luminosity class.

5. Absolute luminosities and intrinsic colours of the Population II Cepheids

The most reliable method to determine the absolute luminosities and intrinsic colours of Population II Cepheids is by way of the globular clusters. Here the reddening due to interstellar absorption and the distance moduli can be derived by other methods. A very extensive investigation in this field has been made by ARP (1955) from photographic two-colour measurements of a number of globular-cluster Cepheids. Later, BAADE and SWOPE (1963), FERNIE (1964) and DICKENS and CAREY (1967) revised Arp's results and reduced them to the *BV* system. Dickens and Carey also added to the material photographic two-colour observations of the Cepheids of the ω Cen cluster. Unfortunately the three revisions of Arp's material led to slightly different results. The discrepancies were due to the use of different mean magnitudes, different methods for the reduction to the *BV* system, and different colour-excesses and distance moduli of the clusters involved.

For the present investigation new determinations of the mean magnitudes and colours have been made for the six globular-cluster variables in table 3 for which three-colour photo-electric observations are available. The results are given in table 4, where the square brackets indicate that the means have been taken over the intensity curve. The differences between the present determinations and the corresponding ones from the two-colour photographic observations are only small.

In table 5 the different determinations of the absolute luminosities and intrinsic colours of the globular-cluster Cepheids have been assembled. Columns 3 and 7 give the data from Fernie's paper, columns 4 and 8 those from Baade and Swope's, while in columns 5 and 9 the determinations by Dickens and Carey have been listed. For the present purpose the absolute luminosities derived by Dickens and Carey have been adopted, with only a few slight corrections due to the new determinations in table 4. In the case of the intrinsic colours, values have been adopted based on Dickens and Carey's revision of Arp's original colours, corrected for interstellar reddening using Fernie's colour excesses.

TABLE 4

Mean magnitudes and colours of those globular-cluster Cepheids which have been observed photo-electrically in three colours

Variable	$\langle V \rangle$	$\langle B \rangle$	$\langle U \rangle$ (mag)	$\langle B-V \rangle$	$\langle U-B \rangle$
M 5 No. 42	11.28	11.82	12.10	+0.59	+0.34
M 5 No. 84	11.36	11.92	12.20	+0.61	+0.33
M 10 No. 2	11.83	12.65	13.05	+0.85	+0.44
M 10 No. 3	12.75	13.58	13.94	+0.84	+0.38
M 80 No. 1	13.42	14.16	14.38	+0.76	+0.24
ω Cen No. 92	13.92	14.49	14.63	+0.57	+0.15

TABLE 5

Luminosities and colours of the globular-cluster Cepheids

Variable	log P	$\langle M_V \rangle$				$(\langle B–V \rangle_{mag})_0$	$(\langle B \rangle - \langle V \rangle)_0$		
		F	B, S	D, C (mag)	adopted	F (mag)	B, S	D, C (mag)	adopted
M 2 No. 1	1.192	−2.5	−2.26	−2.24	−2.24	+0.70	+0.53	+0.55	+0.53
M 2 No. 5	1.244	−2.7	−2.41	−2.39	−2.39	+0.73	+0.55	+0.57	+0.55
M 2 No. 6	1.286	−2.9	−2.62	−2.57	−2.57	+0.68	+0.57	+0.59	+0.57
M 3 No. 154	1.184	−3.4	−2.83	−2.88	−2.88	+0.58	+0.47	+0.51	+0.51
M 5 No. 42	1.410	−3.1	−3.24	−3.25	−3.28	+0.60	+0.49	+0.54	+0.54
M 5 No. 84	1.423	−3.0	−3.19	−3.20	−3.20	+0.66	+0.53	+0.58	+0.56
M 10 No. 2	1.273	−3.0	−2.42	−2.47	−2.47	+0.58	+0.65	+0.65	+0.58
M 10 No. 3	0.896	−2.0	−1.49	−1.54	−1.55	+0.62	+0.59	+0.59	+0.59
M 13 No. 1	0.164	−0.8	−0.46	−0.46	−0.46	+0.60	+0.43	+0.50	+0.45
M 13 No. 2	0.708	−1.7	−1.37	−1.37	−1.37	+0.61	+0.50	+0.56	+0.51
M 13 No. 6	0.325	−0.6	−0.30	−0.30	−0.30	+0.67	+0.54	+0.60	+0.55
M 15 No. 1	0.158	−0.7	−0.46	−0.48	−0.48	+0.31	+0.21	+0.22	+0.24
M 80 No. 1	1.194			−1.76	−1.76			+0.53	+0.54
ω Cen No. 29	1.168			−2.28	−2.28			+0.84	+0.84
ω Cen No. 43	0.063			−0.74	−0.74			+0.35	+0.35
ω Cen No. 48	0.651			−1.41	−1.41			+0.52	+0.52
ω Cen No. 60	0.130			−0.65	−0.65			+0.28	+0.28
ω Cen No. 61	0.357			−0.70	−0.70			+0.52	+0.52
ω Cen No. 92	0.129			−0.17	−0.18			+0.42	+0.46

The adopted values for both quantities are listed in columns 6 and 10 of the table. They have also been plotted against the logarithm of the period in figure 6. In this figure different symbols have been used in order to distinguish between the three major groups: the short-period Cepheids with large amplitudes (crosses), the crested variables (triangles) and the flat-topped variables (open circles).

The plot of intrinsic colour against log P shows several remarkable features. Looking at the short-period variables ($0 < \log P < 0.5$), one notices that the intrinsic colour is redder with increasing period. Leaving out the small-amplitude variable ω Cen No. 92, the six remaining variables give a relation of

$$(\langle B \rangle - \langle V \rangle)_0 = +0.23 + 0.86 \log P. \qquad (1)$$

The line drawn in the figure corresponds to this relation. The individual deviations are large, but most of the discrepancies can be accounted for by incorrect correction for interstellar reddening or by assuming intrinsic colour differences between the variables belonging to different clusters. The colour of the variable ω Cen No. 92 appears to be slightly redder than expected from the relation and also when compared with the other three short-period Cepheids in this cluster.

Except for ω Cen No. 29, the intrinsic colours for all the variables with longer periods appear to be constant and have values between +0.50 and +0.60 mag. This result is surprising where the three variables are concerned with log P between 0.5 and 1.0. Contrary to the expectation at the end of section 3, one should now come to the conclusion that, as far as their colours go, these variables are not related to the group of short-period Cepheids with large amplitudes.

It can be seen from figure 6 that the intrinsic colour of the only crested variable, M 3 No. 154, is slightly bluer than that of the majority of the flat-topped variables, which, excluding ω Cen No. 29, have a mean intrinsic colour of +0.56 mag. However, the difference is small compared with the uncertainty in adopted cor-

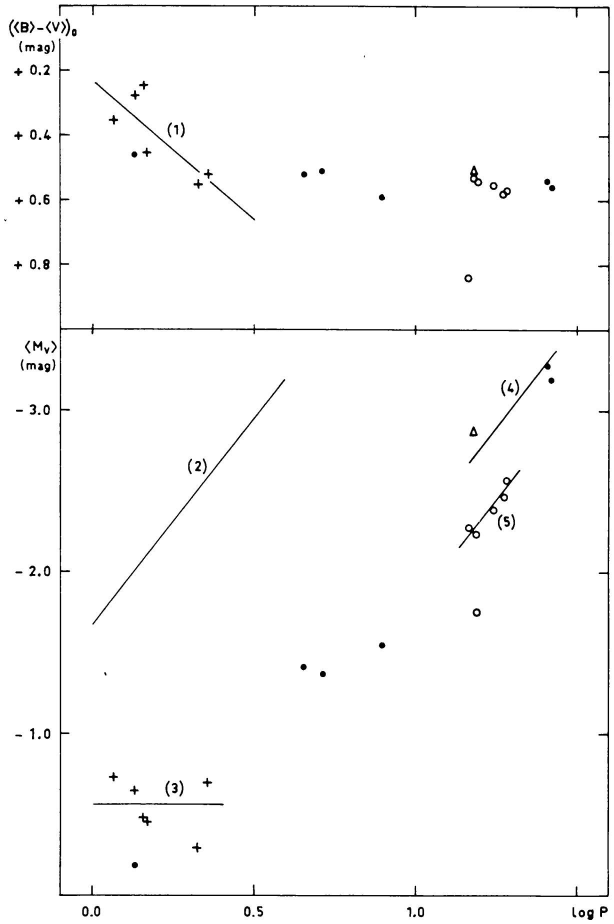

Figure 6. Intrinsic colours and absolute luminosities of the globular-cluster Cepheids. In the upper part of the figure the intrinsic colours are plotted against the logarithm of the period, while the lower part gives the absolute luminosities as a function of the logarithm of the period. Crosses represent the short-period Cepheids with large amplitudes, triangles the crested variables, open circles the flat-topped ones and dots the other Cepheids. Straight lines are drawn representing certain relations used in this investigation. The numbers between parentheses refer to the numbers of the equations, given in the text, for the corresponding relations.

rections for the colour excess. The colour of ω Cen No. 29 turns out to be redder by about 0.3 mag than those of the other flat-topped variables. Dickens and Carey (1967), who measured this colour photographically, already mentioned this, but they have given no further explanation for it.

The mean colour of the two Cepheids in M 5, which have periods of about 26 days, turns out to be +0.55 mag. This value is about the same as for the flat-topped variables and it seems to be in contradiction with the results of sections 3 and 4, where it was found that TW Cap, the galactic Cepheid similar to the two variables in M 5, has an earlier spectral type than W Vir, a galactic flat-topped variable. Again the discrepancy is insignificant in view of the assumptions about the interstellar reddening.

In the luminosity-log P plots of figure 6, the short-period Cepheids with large amplitudes (crosses) can best be characterized by a constant luminosity. Again the scatter is large, but most of it is due to differences from cluster to cluster. As in this case the apparent distance moduli were determined by adopting $M_V = +0.42$ mag for the RR Lyrae variables in the different clusters (Dickens and Carey, 1967), this means that intrinsic differences in luminosity exist between the variables of the different clusters. These differences can be present either in the short-period Cepheids or in the RR Lyrae variables. The mean luminosity resulting from the six short-period Cepheids with large amplitudes is $\langle M_V \rangle = -0.56$ mag. The small-amplitude short-period Cepheid ω Cen No. 92 has a luminosity which is about 0.5 mag fainter than those of the other three short-period Cepheids in ω Cen.

At first glance, the slope of the luminosity-log P relation for the Population I Cepheids adopted by Kraft (1961) is also roughly valid for the globular-cluster Cepheids with periods longer than three days. Only a difference in zero-point of the luminosity of about 2 mag is present. A similar conclusion has also been reached by Arp (1955), Baade and Swope (1963) and Dickens and Carey (1967). A closer inspection, however, shows that, except for M 80 No. 1 which behaves abnormally, the flat-topped Population II Cepheids define a rather strict relation, with practically the same slope as for the classical Cepheids but with a zero-point difference of 2.41 mag. The flat-topped variable M 80 No. 1 is too faint by about 0.5 mag according to this relation. However, part of this discrepancy can be explained by the uncertain position of the RR Lyrae stars in the colour-magnitude diagram of this cluster, as pointed out by Dickens and Carey (1967).

The crested variable M 3 No. 154 is about 0.6 mag more luminous than the flat-topped variables with corresponding periods. Also more luminous are the two M 5 variables with periods of about 26 days. If these two variables are indeed related to the crested variables, as suggested in section 3, this group of variables should be 0.45 mag brighter than the flat-topped variables with corresponding periods.

In the luminosity-log P diagram of figure 6 four straight lines have been drawn to represent the relations adopted for the different groups:

a) for Population I Cepheids, after KRAFT (1961),

$$\langle M_V \rangle = -1.67 - 2.54 \log P, \tag{2}$$

b) for short-period Population II Cepheids with large amplitudes

$$\langle M_V \rangle = -0.56, \tag{3}$$

c) for crested Population II Cepheids

$$\langle M_V \rangle = +0.29 - 2.54 \log P, \tag{4}$$

and

d) for flat-topped Population II Cepheids

$$\langle M_V \rangle = +0.74 - 2.54 \log P. \tag{5}$$

One remark should be made about the variables with log P between 0.5 and 1.0. It is not clear what relation is valid for them. As mentioned in section 3, their light-curves and amplitudes related them to the short-period variables with large amplitudes, but their colours showed more similarity with the variables with longer periods. In view of the scarce material available no conclusions can be made at present.

A discussion of the luminosities of Population II Cepheids will be incomplete without mentioning the investigations in M 31, where BAADE and SWOPE (1963, 1965) found a number of Cepheids of this population type. Unfortunately, for most of the variables only photographic B light-curves are available and furthermore the light-curves are not accurate enough to make divisions into different groups. Of the five Population II Cepheids with periods between 13 and 20 days, which they have found in fields I, III and IV, two variables, IV No. 53 and IV No. 55, are probably flat-topped variables, and one, III No. 170, has probably a crested light-curve. For the remaining two, I No. 113 and I No. 145, no decision can be made from the published light-curves. The difference in apparent B magnitude between III No. 170 on one side and IV No. 53 and IV No. 55 on the other is about 0.6 mag, the crested variable being the brighter. This evidence supports the result derived above concerning the difference in luminosity between the two groups of Population II Cepheids.

The assignment of different period-luminosity relations to the crested and flat-topped Population II Cepheids originated with ARP (1955), who even made a distinction between three different groups. Although the present author is more conservative, the available material of galactic and globular-cluster Population II Cepheids strongly supports the distinction between the two groups.

About the occurrence of the crested and flat-topped variables in the globular clusters, it is remarkable that members of the two groups are never found in the same cluster. A doubtful case might be M 14 where, according to the light-curves published by SAWYER HOGG and WEHLAU (1966), variable No. 1 ($P = 18^d.76$) might be classified as a crested variable and variable No. 7 ($P = 13^d.60$) might be a flat-topped one. However, the differences in their light-curves are not pronounced and the curves are not very accurate. The exclusive presence of members of only one of these two groups of variables in a globular cluster may indicate that the differences between the two kinds of Cepheids are caused by differences in composition. According to PAYNE-GAPOSCHKIN and GAPOSCHKIN (1963), the globular clusters M 3 and M 5, which contain crested variables, are both clusters with an intermediate metal content, while M 2, M 10 and ω Cen, which contain flat-topped variables, have a low metal content. It should also be remembered that the properties of the RR Lyrae variables in these two groups of clusters, such as the mean period of subtype ab, show remarkable differences too.

6. Mean magnitudes, mean colours, colour excesses and distances of the investigated galactic Population II Cepheids

In order to derive the distances of the investigated galactic Population II Cepheids, mean magnitudes and colours are needed in the first place. In table 6 all these quantities are given. They have been derived using the present observations (KWEE and BRAUN, 1967) only. The square brackets without any subscript indicate that the mean has been taken over the intensity curve. In the last column the mean taken over the magnitude curve of the $B-V$ colour has also been computed, in order to relate the two different definitions of the mean colour. The table lists all the galactic Population II Cepheids of the present investigation, with the excep-

TABLE 6

Mean magnitudes and colours of the investigated galactic Population II Cepheids

Name of variable	Type	log P	$\langle V \rangle$	$\langle B \rangle$ (mag)	$\langle U \rangle$	$\langle B \rangle - \langle V \rangle$ (mag)	$\langle U \rangle - \langle B \rangle$	$\langle B-V \rangle$ (mag)	$\langle U-B \rangle$	$\langle B-V \rangle_{mag}$ (mag)
V 527 Sgr	sh	0.100	14.83	15.24	15.32	+0.41	+0.08	+0.444	+0.098	+0.449
VZ Aql	sh	0.223	13.58	14.40	14.91	+0.82	+0.51	+0.848	+0.537	+0.858
V 839 Sgr	sh	0.265	14.89	15.52	15.76	+0.63	+0.24	+0.656	+0.253	+0.664
UY Eri		0.344	11.30	11.76	11.86	+0.46	+0.10	+0.465	+0.098	+0.468
AU Peg		0.380	9.21	10.05	10.64	+0.84	+0.59	+0.846	+0.601	+0.848
V 465 Oph	sh	0.453	13.45	14.34	14.95	+0.89	+0.61	+0.934	+0.647	+0.946
AP Her		1.017	10.78	11.58	12.08	+0.80	+0.50	+0.831	+0.526	+0.838
V 1077 Sgr	cr	1.127	13.01	13.72	14.09	+0.71	+0.37	+0.739	+0.408	+0.748
V 802 Sgr	fl	1.130	13.67	14.77	15.45	+1.10	+0.68	+1.117	+0.751	+1.128
V 410 Sgr	cr	1.140	12.58	13.52	14.14	+0.94	+0.62	+0.960	+0.671	+0.971
CS Cas	cr	1.167	12.05	12.89	13.42	+0.84	+0.53	+0.883	+0.592	+0.904
FI Sct	cr	1.173	14.09	15.60	16.57	+1.51	+0.97	+1.532	+1.02	+1.540
V 1187 Sgr	fl	1.179	13.85	14.88	15.44	+1.03	+0.56	+1.068	+0.625	+1.086
V 741 Sgr	fl	1.182	12.72	13.72	14.36	+1.00	+0.64	+1.024	+0.678	+1.028
CZ Sct	fl	1.188	14.40	15.70	16.60	+1.30	+0.90	+1.342	+0.96	+1.354
AL Sct	fl	1.193	14.02	15.25	16.09	+1.23	+0.84	+1.272	+0.91	+1.290
V 377 Sgr	fl	1.210	13.32	14.32	14.92	+1.00	+0.60	+1.028	+0.621	+1.035
V 478 Oph	fl	1.215	12.94	14.03	14.64	+1.09	+0.61	+1.101	+0.629	+1.118
CO Sct	cr	1.233	14.17	15.67	16.85	+1.50	+1.18	+1.556	+1.27	+1.586
W Vir	fl	1.238	9.93	10.61	10.99	+0.68	+0.38	+0.718	+0.441	+0.730
V 1303 Sgr	cr	1.267	12.88	13.81	14.34	+0.93	+0.53	+0.982	+0.584	+1.000
CC Cyr		1.380	12.02	12.54	12.74	+0.52	+0.20	+0.539	+0.208	+0.546
TW Cap		1.456	10.53	11.12	11.41	+0.59	+0.29	+0.620	+0.312	+0.626

TABLE 7

Colour excesses and distances of the investigated galactic Population II Cepheids

Name of variable	Type	log P	l^{II}	b^{II}	Adopted $(\langle B \rangle - \langle V \rangle)_0$ (mag)	$\langle M_V \rangle$	E_{B-V} (mag)	r (pc)	z	$(\langle U \rangle - \langle B \rangle)_0$ (mag)
V 527 Sgr	sh	0.100	16°.6	−14°.6	+0.32	−0.56	+0.09	10570	−2660	+0.02
VZ Aql	sh	0.223	28.4	− 6.1	+0.42	−0.56	+0.40	3870	− 410	+0.22
V 839 Sgr	sh	0.265	12.7	−12.1	+0.46	−0.56	+0.17	9730	−2040	+0.12
UY Eri		0.344	193.3	−52.6	+0.41	0.00	+0.05	1700	−1350	+0.06
AU Peg		0.380	69.1	−22.3	+0.65	0.00	+0.19	530	− 200	+0.45
V 465 Oph	sh	0.453	25.1	+12.7	+0.62	−0.56	+0.27	4370	+ 960	+0.42
AP Her		1.017	47.1	+ 7.4	+0.55	−2.00	+0.25	2550	+ 330	+0.32
V 1077 Sgr	cr	1.127	16.8	−12.0	+0.50	−2.57	+0.21	9770	−2030	+0.22
V 802 Sgr	fl	1.130	9.1	−11.0	+0.56	−2.13	+0.54	6850	−1310	+0.29
V 410 Sgr	cr	1.140	18.0	−11.4	+0.50	−2.61	+0.44	5940	−1180	+0.30
CS Cas	cr	1.167	113.3	−10.3	+0.50	−2.67	+0.34	5500	− 980	+0.29
FI Sct	cr	1.173	26.5	− 1.9	+0.50	−2.69	+1.01	5620	− 190	(+0.24)
V 1187 Sgr	fl	1.179	4.8	− 6.7	+0.56	−2.25	+0.47	8670	−1010	+0.22
V 741 Sgr	fl	1.182	3.0	− 7.1	+0.56	−2.26	+0.44	5400	− 670	+0.32
CZ Sct	fl	1.188	28.4	− 4.3	+0.56	−2.28	+0.74	7800	− 580	(+0.37)
AL Sct	fl	1.193	19.9	− 6.8	+0.56	−2.29	+0.67	7240	− 850	(+0.36)
V 377 Sgr	fl	1.210	14.5	− 8.3	+0.56	−2.33	+0.44	7350	−1060	+0.28
V 478 Oph	fl	1.215	27.7	+11.9	+0.56	−2.35	+0.53	5500	+1130	+0.23
CO Sct	cr	1.233	22.4	− 3.1	+0.50	−2.84	+1.00	6340	− 340	(+0.46)
W Vir	fl	1.238	319.6	+58.4	+0.56	−2.40	+0.12	2480	+2110	+0.29
V 1303 Sgr	cr	1.267	6.6	− 7.3	+0.50	−2.93	+0.43	8020	−1020	+0.22
CC Lyr		1.380	60.2	+17.3	+0.52	−3.22	+0.00	11170	+3320	+0.20
TW Cap		1.456	29.4	−24.5	+0.55	−3.41	+0.04	5810	−2410	+0.26

tion of TU Cas. This star, as should be remembered, has a variable light-curve and the present material is insufficient to determine the needed quantities. In the second column of the table an indication is given to which of the three major groups of Population II Cepheids the variable belongs: "sh" means short-period with a large amplitude, "cr" means crested and "fl" indicates a flat-topped variable. It should be noted that all the mean U magnitudes fainter than 16 mag are less accurate. So are the corresponding $U-B$ colours in the table.

The intrinsic colours and absolute luminosities of the globular-cluster Cepheids have been discussed in section 5. Taking this discussion as a basis, colour excesses and distance moduli can be derived for the galactic Population II Cepheids investigated at present. Table 7 lists the corresponding data for each of these variables.

The adopted intrinsic colours are given in column 6 of this table. For the short-period variables with large amplitudes they were computed with eq. (1). The intrinsic colours of the flat-topped and of the crested variables were taken constant in both cases, the latter being slightly bluer, according to the results in section 5. AU Peg is a short-period Cepheid with a small amplitude and, therefore, the adopted colour was taken to be 0.10 mag redder than indicated by eq. (1). Such was also the case for the corresponding globular-cluster Cepheid ω Cen No. 92. For TW Cap the adopted value corresponds with the colour of the two variables in the globular cluster M 5.

The other variables have no identical and well-observed counterparts in the globular clusters and therefore the corresponding colour excesses have been estimated from the two-colour diagrams published in this and in an earlier paper (Kwee, 1967b). As a matter of course such a method is a very crude one. The colour excess adopted for UY Eri is +0.05 mag, which fits more reasonably in the two-colour plot of figure 5 than does the value tentatively mentioned in section 4. Also for AP Her a smaller colour excess has been adopted than mentioned in that section. The present value brings the intrinsic colour of this variable at about the same level as of those of the globular-cluster Cepheids M 10 No. 3 and M 13 No. 2. The uncorrected position of CC Lyr in the two-colour diagram (Kwee, 1967b) is such that there is probably no correction for interstellar reddening needed at all for this variable.

The colour excesses in $B-V$ are given in column 8 of table 7. Those for TW Cap and W Vir turn out to be slightly different from the values estimated from direct comparison with globular-cluster Cepheids in section 3. However, the differences are negligible. For AU Peg the resulting colour excess is much smaller than the value of +0.33 mag one can deduce from figure 3 (e) in section 4. However, the latter value was

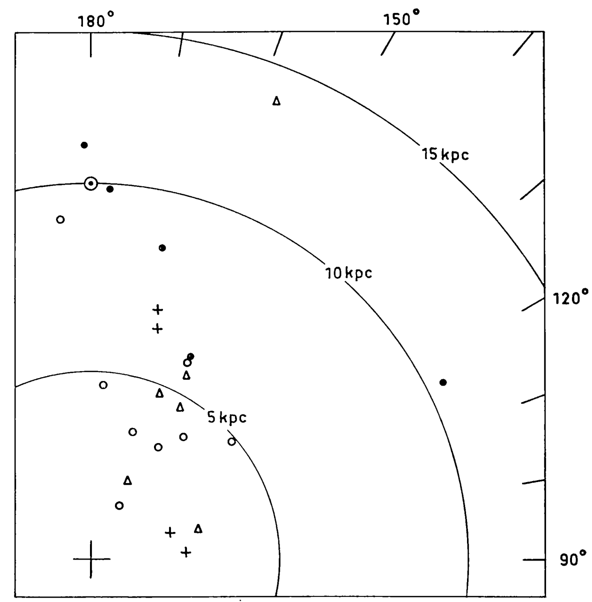

Figure 7. Positions of the investigated galactic Population II Cepheids projected on the galactic plane. The encircled dot in the upper left corner of the diagram represents the position of the Sun and the large cross in the lower left corner the galactic centre. The other symbols are the same as in figure 6.

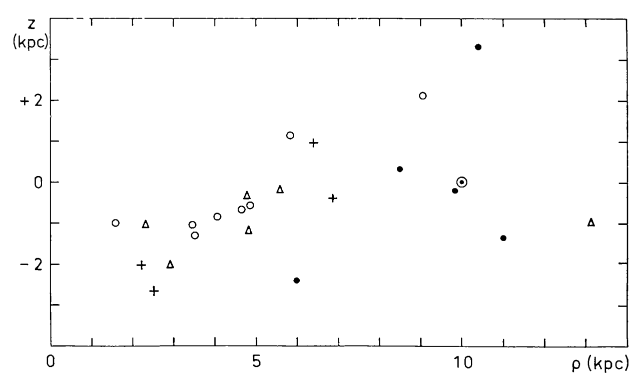

Figure 8. Distances from the galactic plane of the investigated galactic Population II Cepheids. In this diagram the ordinates represent the z distances and the abscissae are the projections on the galactic plane of the distances from the variables to the galactic centre. Symbols used are the same as in figure 6.

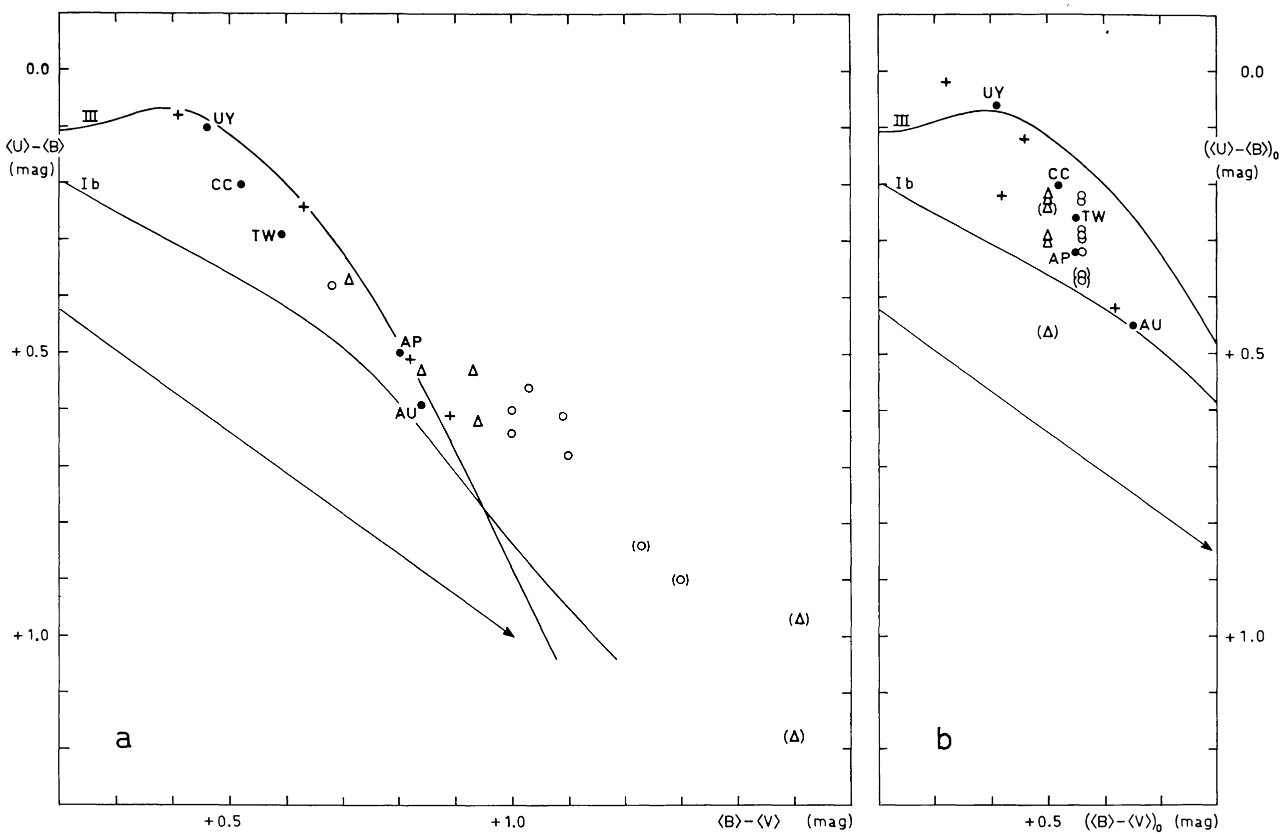

Figure 9. Two-colour diagrams for the mean colours of the investigated galactic Population II Cepheids. Symbols used are the same as in figure 6. Tracks for Ib supergiants and for normal giants as well as the reddening path are indicated in the figure. Symbols between parentheses denote uncertain plots due to large observational errors. In the left part of the figure the colours uncorrected for interstellar reddening have been plotted, while the right part shows the mean colours after applying the adopted corrections.

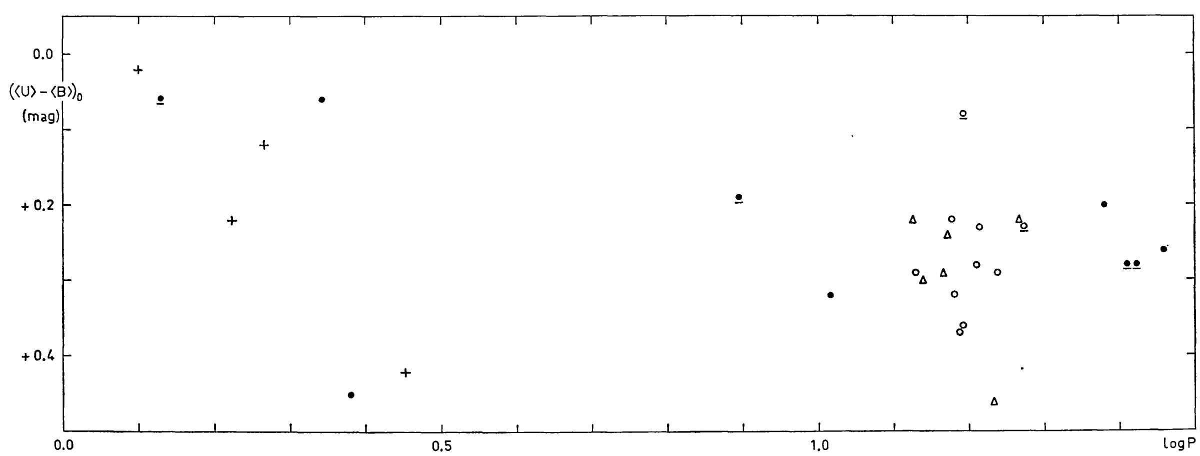

Figure 10. Intrinsic $U-B$ colours of Population II Cepheids as a function of the period. Symbols are the same as used in figure 6. Those with a small horizontal bar underneath refer to the globular-cluster Cepheids.

based on colour-equality with the globular-cluster Cepheid ω Cen No. 92, which, judging from the discussion in section 5, was not a good assumption.

The adopted luminosities for the variables are listed in column 7 of table 7. Here, eqs. (3)–(5) have been used for the corresponding types of Population II Cepheids. Equation (4) was also used for CC Lyr and TW Cap. The luminosities of UY Eri and AU Peg were tentatively assumed to be $\langle M_V \rangle = 0$, while for AP Her a value of -2.0 mag has been adopted.

Assuming the total absorption in V to be three times the colour excess in $B-V$, the distances can now be computed. They are listed in column 9 of table 7, while column 10 gives the distance z above the galactic plane for each of the variables. The positions of the Population II Cepheids projected on the galactic plane are shown in figure 7. In this figure the position of the galactic centre is indicated by the large cross and that of the Sun by the circled dot. The distance between the two has been taken to be 10 kpc. The other symbols are used to distinguish the different groups of Population II Cepheids as before. In figure 8 the distance z is plotted against the projection on the galactic plane of the distance from the variable to the galactic centre. The symbols are the same as in figure 7. It should be emphasized that, owing to the selection of the observed variables, both figures are incomplete and unsuitable for statistical studies. In particular the blank space in the upper left part of figure 8 corresponds with directions towards the constellation Ophiuchus, where the total interstellar absorption is very large. From figure 7 the conclusion can be drawn that the present material apparently does not extend beyond the galactic centre. This conclusion is, of course, greatly dependent on the adopted absolute luminosities of the variables. Figure 8 shows that there is no indication of differences in the degree of concentration to the galactic centre between the different groups of Population II Cepheids.

Finally, assuming the ratio of the reddening in $U-B$ to that in $B-V$ to be 0.72, the intrinsic mean colours in $U-B$ can be computed. They are given in the last column of table 7. Parentheses indicate uncertain values due to large observational errors. Figure 9 shows the apparent and intrinsic two-colour diagrams for the mean colours of the variables. Again the same symbols have been used as before and the tracks of the stars of luminosity classes Ib and III, according to SCHMIDT-KALER (1965), have also been indicated in the plots. Figure 9 (b) shows that in the two-colour diagram the intrinsic colours of practically all the investigated Population II Cepheids fall between the loci of the Ib supergiants and the normal giants. In figure 10 the intrinsic mean $U-B$ colours are also plotted against log P. In this figure the corresponding points for the six globular-cluster Cepheids of table 4 have also been drawn. They have been indicated by a small horizontal bar below the corresponding symbol. The symbols in the figure are the same as used before. The figure shows about the same picture as the $(B-V)$-log P plots in figure 6. For the short-period Cepheids the colours are redder as the periods are larger, while for the variables with longer periods the mean intrinsic $U-B$ colours cluster around $+0.25$ mag. Except for M 80 No. 1 [log $P = 1.194$ and $(\langle U\rangle - \langle B\rangle)_0 = +0.06$ mag], the globular-cluster Cepheids fit in rather well with the galactic Population II Cepheids.

Acknowledgements

The author wishes to express his gratitude to Professor P. Th. Oosterhoff for his continuous interest throughout the investigation. Discussions with other members of the staff of the observatory at Leiden are also gratefully acknowledged.

References

H. A. ABT, 1954, *Ap. J. Suppl.* **1** 63
H. A. ABT and R. H. HARDIE, 1960, *Ap. J.* **131** 155
H. C. ARP, 1955, *Astr. J.* **60** 1
H. C. ARP, 1957, *Astr. J.* **62** 129
W. BAADE and H. H. SWOPE, 1963, *Astr. J.* **68** 435
W. BAADE and H. H. SWOPE, 1965, *Astr. J.* **70** 212
R. J. DICKENS and J. V. CAREY, 1967, *Roy. Obs. Bull.* No. 129
O. J. EGGEN, 1961, *Roy. Obs. Bull.* No. 29
J. D. FERNIE, 1964, *Astr. J.* **69** 258
A. H. JOY, 1949, *Ap. J.* **110** 105
R. P. KRAFT, 1961, *Ap. J.* **134** 616
B. V. KUKARKIN, P. P. PARENAGO, YU. I. EFREMOV and P. N. KHOLOPOV, 1958, *General Catalogue of Variable Stars*, 2nd ed. (Moscow)
B. V. KUKARKIN, YU. I. EFREMOV and P. N. KHOLOPOV, 1960, *First Supplement to the second edition of the General Catalogue of Variable Stars* (Moscow)
K. K. KWEE, 1967a, *Bull. Astr. Inst. Netherlands Suppl.* **2** 97
K. K. KWEE, 1967b, *Bull. Astr. Inst. Netherlands* **19** 260
K. K. KWEE and L. D. BRAUN, 1967, *Bull. Astr. Inst. Netherlands Suppl.* **2** 77
P. TH. OOSTERHOFF, 1960, *Bull. Astr. Inst. Netherlands* **15** 199
C. PAYNE-GAPOSCHKIN and S. GAPOSCHKIN, 1963, *Stars and*

Stellar Systems, ed. G. P. Kuiper and B. M. Middlehurst (Univ. Chicago Press, Chicago) **3** 448
H. B. SAWYER, 1955, *Pub. David Dunlap Obs.* **2** 33
H. SAWYER HOGG and A. WEHLAU, 1966, *Pub. David Dunlap Obs.* **2** 449
TH. SCHMIDT-KALER, 1965 *Landolt-Börnstein Numerical Data and Functional Relationships in Science and Technology*, ed. K. H. Hellwege (Springer-Verlag, Berlin) New Ser., Group VI, Vol. I, section 5.2.2
G. WALLERSTEIN, 1958, *Ap. J.* **127** 583