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COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

Measures of the magnitude of α Ursae minoris on plates taken, mainly by N. W. Doorn, with the "moving camera", by *A. de Sitter*.

The Zeiss photographic twin telescope of the Leiden Observatory has two identical cameras with moving plateholders [1]). The objectives are Zeiss Tessars; aperture 10 cm, focal length 51 cm. On many of the plates, made for general photometric purposes, there has been superposed an exposition of the Polar region. Prof. HERTZSPRUNG suggested to me to measure on these plates the magnitude of Polaris in order to get a new epoch of maximum. The plates used here have all been made in such a way, that for each star a square of the size ·25 mm × ·25 mm is uniformly blackened in 225 exposures of 1 second each. The gratings used in front of the objectives have the spaces between the wires of the same breadth as the thickness of the wires (·2 mm), so that the difference in magnitude between the central image and the first order diffraction image is theoretically ^{m}·98. This value was controlled by the aid of special measures. In the present paper 1^{m}·00 has been used. The 78 sets of two simultaneous plates which I have used have been taken from July 1925 till July 1928, mainly by Mr. DOORN.

I have measured in the Schilt microphotometer on each plate the third order diffraction images of Polaris and the central and first order diffraction images of B. D. 88°4. The opacity of the, somewhat elongated, third order spectra of Polaris lies always between the opacities of the central image and the first order spectra of B.D. 88°4. From these measures in connection with the two extreme readings viz: for fog of the plate and zeropoint, the difference in magnitude between Polaris and B.D. 88°4, apart from a constant, could be derived with sufficient certainty. The mean of each pair of plates taken simultaneously with the two cameras is used as one observation. When the difference between the results of both plates exceeded ^{m}·1, the plates were rejected. From the mostly large deviations of these observations from the final sinecurve we may conclude that their rejection is in general justified. A few plates taken on nights indicated as bad in the observers notebook have been rejected. Thus altogether 17 observations were rejected. The remaining 61 observations have been used for the lightcurve. The observations were arranged according to the phase and divided in 12 groups; 11 of 5 observations, and 1 of 6 observations.

1) See Reports for 1920, 1924, 1926, *B. A. N.* 2, 72, 92.

Table 1 gives in:

column 1 J. D. hel. M. astr. T. Grw. of the middle of each exposure.

2 The Phase, computed by the formula $P = {}^{d-1}$·251994 (J. D. hel. M. astr. T. Grw. — 2420000), corresponding to the period 3^{d}·96835 (GRAMATZKI *A. N.* **217**, p. 453).

3 The difference in magnitude between the mean of the third order images of Polaris and the central image of BD 88°4.

4 The difference in magnitude between the results of the two plates.

5 The deviation from the sinecurve. Those of rejected observations are enclosed in parentheses.

Table 2 gives in:

column 1 The mean of the phases of each group of 5 obs. (the last of 6 obs.).

2 The mean of the observed mag of each group.

3 The deviation from the sinecurve.

TABLE 1.

J. D.	*P*	*m*	Δ	*O—C*	J. D.	*P*	*m*	Δ	*O—C*	J. D.	*P*	*m*	Δ	*O—C*
		m	m	m			m	m	m			m	m	m
2425309·656	·001	·15	·02	— ·03	2425152·301	·349	·24	·14	— ·10	2424907·624	·692	·26	·13	— ·02
4341·469	·024	·13	0	— 6	5152·341	·359	·24	3	— 10	4907·658	·700	·16	8	— 12
5321·671	·029	·16	4	— 3	5124·566	·360	·18	1	(— 17) [2]	4919·565	·701	·30	5	(+ 2) [4]
4345·468	·032	·14	4	— 5	5124·599	·368	·29	8	(— 6) [2]	4907·704	·712	·24	6	— 3
4345·537	·049	·15	4	— 5	5152·389	·371	·33	10	— 2	5439·516	·725	·24	3	— 3
5123·370	·058	·20	2	0	5152·434	·382	·30	5	— 5	5304·679	·747	·24	3	— 1
4619·394	·060	·16	3	— 4	5275.480	·389	·46	10	+ 11	4352·434	·787	·22	0	— 1
5123·419	·071	·16	0	— 4	5180·322	·410	·18	5	(— 18) [3]	5245·332	·792	·33	16	(+ 10)
5123·478	·085	·20	4	— 1	4406·506	·413	·38	6	+ 2	4352·492	·802	·33	30	(+ 11)
4639·412	·104	·14	1	— 8	5180·378	·424	·20	9	(— 16) [3]	5245·380	·804	·31	10	+ 9
4921·328	·145	·26	4	+ 2	4922·497	·440	·48	5	(+ 12)	4848·573	·811	·20	9	— 2
4905·490	·154	·22	1	— 2	5299·605	·469	·44	3	+ 8	4352·531	·812	·14	12	(— 6)
4921·379	·158	·18	1	— 6	5299·644	·478	·40	1	+ 4	5245·420	·814	·32	5	+ 10
4921·422	·169	·24	7	— 1	4851·297	·498	·33	8	— 3	5257·347	·820	·28	8	+ 7
4159·510	·172	·37	6	(+ 12) [1]	4851·339	·508	·41	14	(+ 5)	5257·385	·829	·28	7	+ 7
4921·527	·195	·28	1	+ 2	4978·478	·547	·41	2	+ 6	5261·420	·846	·28	3	+ 8
5298·551	·203	·42	20	(+ 15)	5442·844	·564	·28	1	— 6	4908·329	·869	·16	9	— 4
4921·605	·215	·26	1	— 1	4851·582	·570	·28	3	— 6	4638·491	·872	·11	0	— 9
5298·627	·222	·36	1	+ 8	5442·887	·575	·31	4	— 6	4920·303	·887	·28	3	+ 9
4961·324	·224	·16	12	(— 12)	4851·664	·590	·29	2	— 4	4920·339	·897	·24	6	+ 5
4921·664	·230	·36	3	+ 8	4879·490	·602	·26	5	— 7	5273·560	·905	·22	1	+ 3
5298·661	·231	·28	3	0	4879·530	·612	·31	0	— 1	4920·386	·908	·24	9	+ 5
4961·368	·235	·26	4	— 3	4879·576	·624	·27	14	(— 5)	4920·465	·928	·23	0	+ 5
4346·507	·294	·32	14	(0)	5165·298	·624	·14	12	(— 18)	4920·523	·942	·24	0	+ 6
4620·480	·333	·44	2	+ 10	4919·438	·669	·38	14	(+ 11) [4]	4920·558	·951	·22	1	+ 4
4894·309	·337	·37	4	+ 2	4907·580	·681	·32	9	+ 3	5301·655	·985	·13	4	— 5

1) Exposure on a rather clear moment of a cloudy night.
2) When leaving the dome Mr. Doorn saw a great many small clouds near the pole.
3) "It is rapidly getting foggy and moisty".
4) Drifting clouds near the pole.

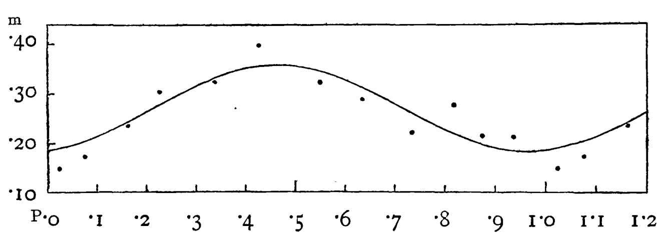
TABLE 2.

P	m	O — C
·0237	·148	— ·041
·0761	·173	— ·031
·1643	·236	— ·008
·2266	·304	+ ·026
·3368	·326	— ·006
·4262	·399	+ ·043
·5508	·324	— ·022
·6345	·287	— ·026
·7342	·220	— ·040
·8194	·277	+ ·060
·8746	·215	+ ·018
·9365	·212	— ·027

From the data of Table 2 the sinecurve

$$m = {\cdot}2710 + {\cdot}0876 \sin(\varphi - 77^\circ{\cdot}3),$$

where $\varphi = 2\pi P$, was determined by a least square solution. The mean error of one observation, as derived from the deviations of the individual observations from this sinecurve, is $\pm\,^{\mathrm{m}}{\cdot}058$. The mean error of the epoch is found to be $\frac{\mu}{\sqrt{\frac{1}{2}n{\cdot}a}} \cdot \frac{P}{2\pi} = \pm\,^{\mathrm{d}}{\cdot}076$, where μ



is the mean error of one observation, n is the number of observations, a is the total amplitude. The maximum is found to occur at the phase $^{\mathrm{P}}{\cdot}4674$ or $1^{\mathrm{d}}{\cdot}844$. The epoch of maximum nearest to the mean epoch of the observations is J. D. hel M. astr. T. Grw. 2424894·819 $\pm\,^{\mathrm{d}}{\cdot}076$. (m. e.) The data of Table 2 are represented in the diagram.

The epochs used for a least square solution for the period are:

	J. D.	E	n	a	μ	p	$O-C$
Müller & Kemf	2407696·57	0	305	^{m}·056	± ^{m}·077	162	— ·02
Edw. C. Pickering	8228·45	134	10	^{m}·078	± ^{m}·033	56	+ ·13
Pannekoek [1])	13045·81	1348	510	^{s}·92	± ^{s}·84	615	+ ·15
King	7791·79	2544	98	^{m}·119	± ^{m}·055	457	+ ·23
Hertzsprung [2])	8985·86	2845	20	^{m}·171	± ^{m}·017	2020	— ·12
Stebbins	9299·15	2924	17	^{m}·078	± ^{m}·021	232	— ·31
Gramatzki	23954·08	3845	$68\frac{1}{2}$	^{m}·126	± ^{m}·051	420	— ·05
Bottlinger	3620·77	4013	16	^{m}·146	± ^{m}·027	468	— ·01
A. de Sitter	4894·82	4334	61	^{m}·175	± ^{m}·058	546	+ ·27

The epochs of MÜLLER, PICKERING and PANNEKOEK have been taken from PANNEKOEK *Proc. Amst. Ac.* 1913, p. 1192. The observations used by KING are to be found in *H. A.* **59** p. 250 table IV. The means of the 14 groups of 7 observations each are

Corrected phase

p	m	$O-C$
·0386	— ·11	— ·05
·1314	— ·03	+ ·02
·2086	— ·05	·00
·2529	— ·05	— ·01
·3300	— ·01	·00
·4500	+ ·05	+ ·02
·5229	+ ·04	— ·01
·6300	+ ·07	+ ·01
·6800	+ ·03	— ·02
·7143	+ ·07	+ ·02
·7529	+ ·02	— ·01
·8043	— ·02	— ·04
·9129	+ ·04	+ ·06
·9729	— ·04	·00

$P = {}^{d-1}$·251995 (J. D. — 2400000)
The sinecurve derived from these figures is

$$m = \cdot 4217 - \cdot 0338 \sin\varphi - \cdot 0488 \cos\varphi.$$

To this corresponds the maximum 2417791·79.
The mean error of one observation as derived from the deviations of the individual observations from the sinecurve is ± ^{m}·055.

HERTZSPRUNG'S epoch is taken from *A. N.* **189** p. 89. Prof. HERTZSPRUNG called my attention to the fact that the mean error of the epoch there given is too high. It should be ± ^{d}·042 instead of ± ^{d}·083.

[1]) The two series combined. The series of 259 observations gives an amplitude of ^{s}·90, the series of 251 observations gives an amplitude of ^{s}·94. For computing the relative weight of both combined I have used as the amplitude ^{s}·92.

[2]) In this case the normal points are used instead of the individual observations for computing the relative weight. When using the individual observations the calculated weight would be found higher.

STEBBINS' observations (*A. N.* **192**, p. 189) are:

p	m	$O-C$
·082	·102	— ·035
·091	·147	+ ·011
·134	·179	+ ·047
·230	·137	+ ·002
·246	·120	— ·017
·337	·178	+ ·024
·349	·142	— ·015
·350	·170	+ ·013
·523	·195	— ·005
·576	·191	— ·018
·604	·219	+ ·007
·607	·210	— ·003
·675	·226	+ ·011
·748	·212	+ ·003
·832	·169	— ·025
·860	·196	+ ·009
·923	·162	— ·008

m. e. = ± ^{m}·021

$P = {}^{d-1}$·2520098 (J. D. — 2418983·337).
The sinecurve derived from the figures is

$$m = \cdot 1733 - \cdot 0357 \sin\varphi - \cdot 0220 \cos\varphi$$

To this corresponds the maximum 2419299·15.

GRAMATZKI gives in *A. N.* **217**, p. 453 two series of observations with a new type of visual photometer. The square of the mean error of one observation of the second series, as derived from the differences between the observations in phase next to each other, is about twice as large as the square of the mean error of one observation of the first series.

The observations of the second series were therefore given half weight. All observations were then arranged according to phase and divided into 14 groups of about equal weight.

p	m	$O-C$
·0397	·188	— ·013
·0970	·223	+ ·041
·1276	·181	+ ·008
·2433	·155	+ ·001
·3039	·113	— ·043
·3536	·200	+ ·037
·4178	·165	— ·015

P	m	O—C
·5396	·195	—·028
·5840	·275	+·037
·6870	·254	—·010
·7798	·297	+·027
·8071	·269	+·001
·8436	·268	+·006
·9127	·198	—·048

$P = {}^{d-1}{\cdot}2520098$ (J. D. — 2422903·458).

The sinecurve derived from these figures is

$$m = {\cdot}2121 - {\cdot}0584 \sin\varphi + {\cdot}0037 \cos\varphi$$

To this corresponds the maximum $2422954^{d}{\cdot}082$. The mean error of one observation as derived from the deviations of the individual observations from the sinecurve is $\pm {}^{m}{\cdot}051$.

The epoch of BOTTLINGER was derived from the observations which Dr. BOTTLINGER kindly sent to us, and for wich I express my thanks. There are 5 observations with one photoelectric cell and 11 with another. Assuming a constant difference of ${}^{m}{\cdot}026$ between the results obtained with the two cells, the observations, arranged according to phase and corrected for this difference, are:

P	m	O—C
·079	·339	—·001
·127	·313	— 6
·150	·296	— 13
·284	·243	— 10
·297	·235	— 14
·305	·268	+ 21
·342	·224	— 14
·570	·313	+ 48
·570	·268	+ 3
·632	·267	— 24
·787	·353	— 2
·796	·374	+ 18
·797	·313	— 46
·839	·374	+ 5
·916	·350	— 26
·986	·431	+ 62

m. e. $= \pm {}^{m}{\cdot}027$

$P = {}^{d-1}{\cdot}251994$ (J. D. — 2420000)

$$m = {\cdot}3038 + {\cdot}0386 \sin\varphi - {\cdot}0616 \cos\varphi$$

To this corresponds the maximum 2423620·766.

A least square solution of these epochs with relative weights computed by the formula $p = \frac{na^2}{\mu^2}$, where n is the number of observations, a the total amplitude, and μ the mean error of one observation, gives for the period $3^{d}{\cdot}968148 \pm {\cdot}000055$ (m. e.)

The formula found here for the epoch of maximum light of Polaris is thus

Max.=J.D. hel. M. astr. T, Grw. $2424894{\cdot}65 + 3^{d}{\cdot}968148\,E$
(m. e.) $\pm\ {\cdot}06 \pm\ {\cdot}000055$

I wish to thank Prof. HERTZSPRUNG for his advice and help during my work.