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COMMUNICATIONS FROM THE OBSERVATORY AT LEIDEN

Photovisual photometry of the eclipsing variable μ_1 Scorpii, by *H. van Gent.*

1. *Introduction.*

The star μ_1 Scorpii (= GC 22677 = HD 151890, $16^h45^m{\cdot}1$ — $37°51'$, 1900, $3^m{\cdot}09$, spectrum B3p) was found to be a spectroscopic binary with double lines by SOLON I. BAILEY[1]) in 1896 on objective prism plates taken with the 13-inch Boyden telescope at the Harvard Southern Station, Arequipa, Peru.

In 1920 the star was announced as a variable of the eclipsing type by Miss A. MAURY[2]), who also gave two preliminary light curves derived from estimates of the relative intensities of the continuous spectra of μ_1 and μ_2 Scorpii made independently by herself and by Miss LEAVITT on BAILEY's Arequipa plates. Although these light curves are not accurate enough for a computation of the geometric and orbital elements for this binary system, its β Lyrae character is already clearly indicated.

A spectroscopic orbit was computed, also by Miss MAURY[3]), from radial velocity measures on BAILEY's plates covering the interval from 1892 till 1918. No comparison spectra being present on these plates, only the relative orbit could be derived. As this orbit shows some eccentricity and as the β Lyrae character of the preliminary light curve shows the components to be nearly in contact, LUYTEN[4]) thought the system might be a favorable case for detecting motion of the periastron if the value for the eccentricity in the spectroscopic orbit e = ·05 could be trusted. By dividing the Harvard-Arequipa radial velocities already mentioned into two groups, for which he derives separate spectroscopic orbits, LUYTEN finds some indication for regression of the periastron in a period which is very roughly estimated at 120 years. It is pointed out by him, however, that something more certain about the periastron rotation will only become known after further observations, both photometric and spectroscopic, will have been made.

Following this suggestion independently, photometric observations have been made by RUDNICK and ELVEY[5]) at the MacDonald Observatory, Fort Davis, Texas, with a photoelectric cell attached to the 12-inch Yerkes refractor, and by the writer at the Union Observatory, Johannesburg, South Africa, by means of photovisual observations with the 10-inch Franklin Adams telescope.

2. *Plate material, measures and reduction.*

The observational material consists of a series of 85 plates taken on 19 nights in 1936 from July 1 till October 31 and an isolated plate on 1937 May 3. The plates have been obtained with the same instrumental equipment as used for observations on V Puppis described in *B.A.N.* No. 317. The star μ_2 Scorpii (= GC 22691 = HD 151985, $16^h\ 45^m{\cdot}6$ — $37°51'$, 1900, $3^m{\cdot}64$, Sp B2) provides an almost ideal comparison star. As its distance from the variable is only $5'{\cdot}8$ or 1·9 mm on the plates obtained it was not necessary to reverse the plate by 180° for elimination of errors from unequal sensitivity across the plate. As the comparison star and the variable have practically the same spectral type, no effects from difference in colour are to be feared either. An objective grating ($d = l;\ d + l$ = 1·900 mm) has been used in order to obtain the magnitude scale. The plates employed are all of the brand *Eisenberger Ultrarapid hochfarbenempfindlich.* The effective wavelength, derived from measures of the distance between the two first order grating images on the plate, is 5604 Å.

Usually a plate contains 12 groups of 8 exposures each. The exposure time being 10 seconds and the time lost between two consecutive exposures 5 seconds, exposures were thus made at the rate of four a minute.

The plates have been measured with the old thermopile photometer[6]) of the Leiden Observatory by

1) *H.C.* No. 11.
2) *H.A.* **84**, p. 168.
3) L.c.
4) *Publ. Minnesota Obs.* **2**, p. 37.
5) *Ap. J.* **87**, p. 553.
6) For description see *B.A.N.* No. 60.

Mr. C. J. KOOREMAN, who also took care of the reduction of the measures. The galvanometer readings have been converted into provisional magnitudes with the aid of a table constructed by A. J. WESSELINK[1]). To these provisional magnitudes the formula $\frac{\Sigma_v - \Sigma_c}{\Delta_v + \Delta_c}$ was applied, Σ_v and Σ_c being the sums of the provisional magnitudes of first order spectrum and central image for variable and comparison star respectively, and Δ_v and Δ_c their differences. By this procedure the difference between the magnitudes of variable and comparison star is expressed in the difference between central image and first order spectrum as unit. For a grating the bars of which are of the same width as the spaces between them this difference is theoretically $^{m}\cdot981$. Consequently all results from the formula have been multiplied by this quantity in order to obtain the correct magnitude differences.

For a number of plates the first order spectra of μ_2 Scorpii were too faint to yield a reliable measure. In these cases the difference in brightness between μ_1 and μ_2 Scorpii has been derived from the provisional magnitudes by linear interpolation, with the application of a small correction. This correction was found by comparing for a few plates with complete sets of measurable images the results obtained by the reduction first mentioned and by linear interpolation.

3. *Light curve, orbital eccentricity and period.*

The time of the middle of each exposure has been converted into Julian Day Heliocentric Mean Time Greenwich. Only the mean for each plate has been given in Table 1, 2nd column. For the construction of the light curve Miss MAURY's period, viz. $1^{d}\cdot44627$ has been used. The phases, in column 3, have been computed with the reciprocal of this period, according to the formula: phase = (J.D. — 2420000) × $^{d-1}\cdot691434$. Again only the plate mean for the difference in brightness between μ_1 and μ_2 Scorpii has been given in column 4, the number of images used in this mean being indicated in column 5.

By plotting brightnesses against phases the light curve represented by Figure 1 results. When we compare this light curve with the one given by RUDNICK and ELVEY[2]) the agreement between the two appears to be not quite satisfactory, although both have been made from material of the same opposition, 1936. For both light curves the ranges for primary and secondary minimum are $^{m}\cdot30$ and $^{m}\cdot19$ respectively. There is, however, a small systematic difference, independent of phase, between RUDNICK and ELVEY's observations and the author's,

1) *B.A.N.* No. 318.
2) L.c.

TABLE 1.

plate	J.D. Hel. M.T.Grw.	phase	brightness	n	plate	J.D. Hel. M.T.Grw.	phase	brightness	n	plate	J.D. Hel. M.T.Grw.	phase	brightness	n
	2420000+		m			2420000+		m			2420000+		m	
12456	8351·2587	·3442	—·573	47	12577	8381·2206	·0609	·571	85	12740	8408·3040	·7873	·566	94
12457	8355·2490	·1032	·599	93	12578	·2388	·0734	·578	96	12742	8410·2332	·1212	·600	84
12460	·2984	·1374	·624	94	12579	·2568	·0859	·582	95	12745	8414·2845	·9224	·345	88
12461	·3162	·1497	·616	96	12580	·2751	·0986	·581	88	12746	·3023	·9347	·337	96
12462	·3349	·1626	·606	96	12581	·2928	·1108	·601	95	12747	·3203	·9471	·356	96
12463	·3529	·1751	·618	96	12584	8383·2542	·4670	·454	92	12748	·3383	·9596	·355	96
12464	·3710	·1876	·626	95	12585	·2719	·4792	·481	94	12749	·3563	·9720	·402	96
12465	·3889	·2000	·629	96	12586	·2898	·4916	·487	92	12750	·3742	·9844	·425	94
12466	8356·2525	·7971	·566	95	12587	·3079	·5041	·525	96	12752	8415·2427	·5849	·614	95
12467	·2712	·8100	·571	96	12588	·3260	·5166	·528	95	12753	·2610	·5976	·592	88
12468	·2892	·8225	·524	95	12589	·3442	·5292	·536	91	12754	·2738	·6064	·606	40
12469	·3071	·8348	·536	94	12590	·3620	·5415	·549	94	12907	8441·3161	·6129	·618	96
12470	·3253	·8474	·516	94	12591	·3772	·5520	·541	64	12908	·3342	·6254	·607	95
12471	·3432	·8598	·486	96	12592	·3925	·5626	·574	92	12917	8447·2216	·6962	·657	92
12472	·3702	·8785	·426	96	12593	·4105	·5750	·580	94	12918	·2397	·7087	·656	94
12473	·3875	·8905	·406	96	12602	8387·3396	·2918	·588	95	12919	·2576	·7211	·630	91
12474	·4055	·9029	·369	96	12603	·3576	·3042	·571	95	12920	·2757	·7336	·626	93
12475	·4228	·9148	·326	92	12604	·3757	·3168	·561	93	12959	8457·2628	·6390	·630	96
12480	8358·2989	·2121	·628	94	12605	·3925	·3284	·558	80	12960	·2808	·6515	·648	96
12481	·3169	·2245	·638	96	12606	·4116	·3416	·550	95	12961	·2960	·6620	·660	64
12482	·3349	·2369	·618	96	12729	8406·2265	·3508	·542	96	12988	8472·2360	·9920	·466	108
12483	·3529	·2494	·596	96	12730	·2445	·3633	·522	96	12989	·2555	·0055	·497	90
12484	·3708	·2618	·599	94	12731	·2625	·3757	·501	96	12990	·2680	·0141	·521	33
12485	·3889	·2743	·595	95	12732	·2805	·3881	·474	95	12991	8473·2171	·6704	·633	99
12486	·4068	·2866	·587	94	12733	·2986	·4006	·453	94	12992	·2358	·6833	·645	88
12572	8380·2776	·4088	·462	96	12734	8407·2139	·0335	·511	97	13461	8657·4092	·0270	·518	97
12573	·2956	·4213	·462	96	12735	·2320	·0461	·531	96					
12574	·3136	·4337	·448	94	12737	8408·2458	·7470	·592	96					
12575	·3316	·4462	·438	96	12738	·2641	·7597	·568	92					
12576	8380·3496	·4586	·458	95	12739	·2832	·7729	·570	96					

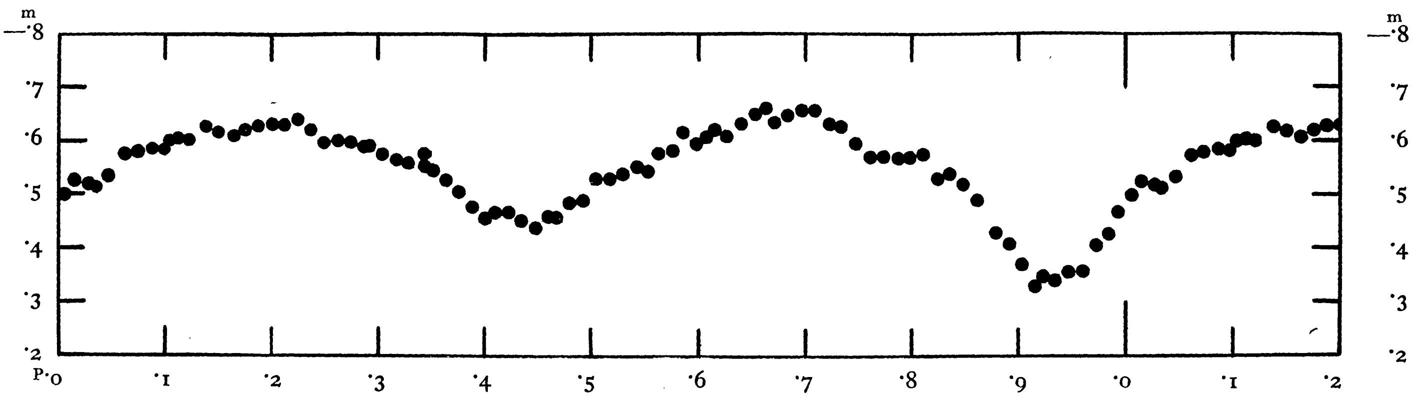

FIGURE 1.

the author making the variable slightly brighter with respect to μ_2 Scorpii than RUDNICK and ELVEY do. The amount of this difference was determined to be ^{m}·022 ± ^{m}·002. Although the difference found seems real, its explanation is uncertain. It might be caused by a small difference in colour between μ_1 and μ_2 Scorpii together with a difference between the effective wavelengths in which the two photometric studies have been made. In the *Draper Catalogue* the spectral type of the variable is given as B3p, one tenth of a spectral class later than that of the comparison star. Photovisually the variable might therefore be expected to be about ^{m}·03 brighter with respect to the comparison star than photographically, so that the effect found is in the right direction and also is of the correct order of magnitude. As no information is given by RUDNICK and ELVEY about the spectral sensitivity of their equipment, nothing further can at present be said about this difference.

By trying to superpose the two light curves another small difference was found between them. Near begin and end of the eclipses RUDNICK and ELVEY's curve shows slightly brighter values than the author's, their maxima therefore appearing flatter. Consequently in the orbit computation they find smaller values for the ellipticity constant and for the ratio of the surface brightnesses than the author.

The light curve does not show any sensible orbital eccentricity, the minima appearing to be symmetrical and equally spaced. By the method described in *B.A.N.* No. 147 by E. HERTZSPRUNG sharp determinations were made of the phases of the lines of symmetry for both minima. The results are:

phase of primary minimum ·9316 ± ·0015 (m.e.)
phase of secondary minimum ·435 ± ·003 (m.e.)

The difference in phase between secondary minimum and the point midway between two consecutive primary minima consequently is:

$$D - \tfrac{1}{2} = \frac{2\,e\cos\omega}{\pi} = {\cdot}003 \pm {\cdot}003 \text{ (m.e.)}.$$

If we compute this quantity from the figures given for e and ω by Miss MAURY[1]), we find:

$$D - \tfrac{1}{2} = \frac{2\,e\cos\omega}{\pi} = {\cdot}031.$$

This latter value is certainly excluded by the observations, so that we find that either the value for ω is at present very nearly 90° or 270°, or that the spectroscopic value for e is spurious. Although a value of 90° for ω would agree with LUYTEN's estimate for the period of rotation of the line of apsides, the author favours the second alternative, as it seems to be certain that in spectroscopic orbits systematic errors in the radial velocities often cause erroneous values for the eccentricity. This is the more likely as Miss MAURY states explicitly that the lines of μ_1 Scorpii are mostly hazy, asymmetrical and difficult to measure.

As is well known orbital eccentricity may also be revealed by a difference in width between the two minima when the value for ω is near 90° or 270°. This difference however, which is proportional to $e \sin \omega$, is a much poorer criterion for orbital eccentricity than the position of secondary minimum between two consecutive primary minima. Especially in a case with shallow minima like μ_1 Scorpii results from application of this criterion are most uncertain. Therefore the author refrained from analysing the light curve for presence of $e \sin \omega$ in the difference between the widths of the two minima.

Consequently in the present discussion the orbit is considered to be circular.

As it is of importance to have an epoch of an observed minimum as sharp as possible from the present material for future investigations about this star, the following normal epoch of principal minimum is derived from our observations:

1) L.c.

J.D. $2428414^{d}\cdot2978 \pm {}^{d}\cdot0022$ (m.e.).

From Miss MAURY's radial velocities the following normal epoch of zero relative radial velocity, corresponding to principal minimum in the light curve, was derived:

J.D. $2415591^{d}\cdot6830 \pm {}^{d}\cdot0027$ (m.e.).

The number of periods elapsed between these epochs is 8866; consequently the following improved value of the period can now be given:

period $= 1^{d}\cdot4462683 \pm {}^{d}\cdot0000004$ (m.e.).

The uncertainty of the period is of the order of ·04 seconds.

4. *Determination of orbital and geometric elements.*

As the light curve bears considerable weight (about 500 000 m^{-2}) it was made the basis of a new determination of the fundamental quantities in this important eclipsing system. A solution for uniform discs was made, assuming two similar three-axial ellipsoids with their longest axes in a line, as is customary in such cases. The orbit was considered to be circular.

The phase (by the formula already mentioned: phase = (J.D. − 2420000) × $^{d-1}\cdot691434$) of mid-primary eclipse in the light curve was determined by least squares from all observations to be at ·9327. For all plates the phase counted from mid-primary eclipse was computed; these data are presented in Table 2, column 2. Phase has been plotted against brightness in Figure 2. Column 3 gives the quantity cos 2ϑ, ϑ being the system's anomaly, also counted from mid-primary eclipse.

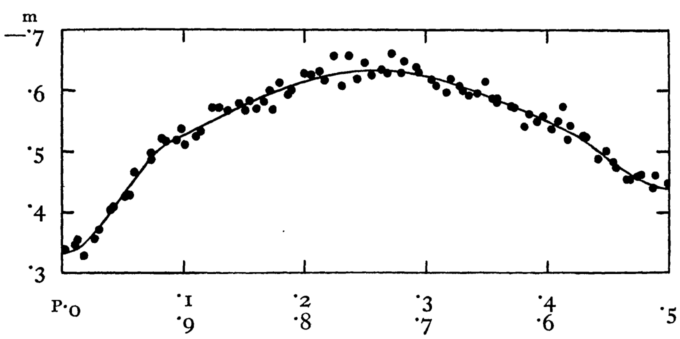

FIGURE 2.

The intensity of light l, expressed in the maximum light as unit, is connected to the eccentricity of the equatorial section ε by the well known formula:

$$l^2 = 1 - \varepsilon^2 \sin^2 i \cos^2 \vartheta,$$

where i denotes the orbital inclination.

Therefore l^2 was computed for each plate from the brightness given in Table 1 and plotted against $\cos 2\vartheta = 2\cos^2\vartheta - 1$. The result is shown in Table 2, column 4, and in Figure 3, dots denoting primary

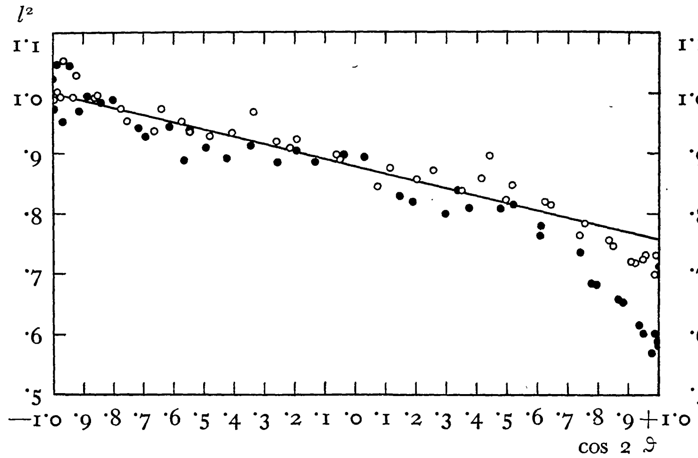

FIGURE 3.

minimum and open circles secondary minimum. The relation between the coordinates should be linear outside the eclipses. From an inspection of the diagram the eclipse was considered not to start before cos 2 ϑ = + ·6, and a least squares solution was made for a straight line through the points to the left of this phase. Its result is:

$$l^2 = + \cdot8781 - \cdot1219 \cos 2\vartheta$$
$$\pm \cdot0083 \text{ (m.e.)}.$$

This straight line has been drawn in Figure 3. Accordingly:

$$\varepsilon^2 \sin^2 i = + \cdot2438 \pm \cdot0165 \text{ (m.e.)}.$$

As it seems that the brightness of the variable is greater just outside the secondary eclipse than at the same phase outside the primary eclipse, some effect from reflection was suspected. Therefore the differences in intensity of light l as observed and as computed from the ellipticity of the components, given in Table 2, column 6, have been analysed for the presence of a periodic term depending on cos ϑ. The result of a least squares solution was, if we consider only points outside the eclipses:

$$l_{obs.} - l_{comp.} = -\cdot00026 - \cdot01151 \cos \vartheta$$
$$\pm\cdot00155 \pm\cdot00145 \text{ (m.e.)}.$$

The amount of reflection as computed from this formula has been given in Table 2, column 7, and the differences with the quantities in column 6 are presented in column 8. In this way rectification has been performed as well for ellipticity as for reflection, so that column 8 represents the rectified intensity curve. Figure 4 was obtained by plotting the values $l_{obs.} - l_{ell.}$ against cos ϑ; the straight line drawn in the diagram represents the relation for the reflection just found. As part of the reflecting area of one component is covered by the other one during eclipse, the amounts of reflection during eclipse as computed from the formula cannot be regarded as strictly correct and are slightly too high.

TABLE 2.

plate	phase from minimum	cos2ϑ	$l^2_{obs.}$	$l^2_{ell.}$	$l_{obs.}-l_{ell.}$	reflection	$l_{obs.}-l_{ell.}+refl.$	brightness from maximum		O-C
								obs.	comp.	
								m	m	m
12746	·0020	+ ·9997	·5800	·7563	—·1081	—·0118	—·0963	+·296	·300	—·004
12745	·0103	·9916	·5886	·7572	—·1030	·0117	—·0913	·288	·295	—·007
12747	·0144	·9837	·6006	·7582	—·0957	·0117	—·0840	·277	·291	—·014
12475	·0178	·9751	·5684	·7593	—·1175	·0117	—·1058	·307	·286	+·021
12748	·0269	·9434	·5995	·7631	—·0993	·0116	—·0877	·278	·268	+·010
12474	·0298	·9307	·6152	·7647	—·0902	·0116	—·0786	·264	·262	+·002
12749	·0393	·8805	·6538	·7708	—·0694	·0114	—·0580	·231	·237	—·006
12473	·0422	·8627	·6585	·7730	—·0677	·0114	—·0563	·227	·229	—·002
12750	·0517	·7963	·6821	·7811	—·0581	·0112	—·0469	·208	·202	+·006
12472	·0542	·7769	·6833	·7834	—·0585	·0111	—·0474	·207	·195	+·012
12988	·0594	·7341	·7355	·7886	—·0304	·0110	—·0194	·167	·180	—·013
12989	·0728	·6099	·7787	·8038	—·0141	·0106	—·0035	·136	·145	—·009
12471	·0729	·6089	·7631	·8039	—·0230	·0106	—·0124	·147	·144	+·003
12990	·0815	·5198	·8139	·8148	—·0005	·0103	+·0098	·112	·123	—·011
12470	·0852	·4796	·8065	·8197	—·0073	·0102	+·0029	·117	·120	—·003
13461	·0944	·3751	·8095	·8324	—·0127	·0098	—·0029	·115	·111	+·004
12469	·0978	·3351	·8368	·8373	—·0002	·0097	+·0095	·097	·108	—·011
12734	·1009	·2982	·7991	·8418	—·0236	·0095	—·0141	·122	·105	+·017
12468	·1102	·1850	·8184	·8556	—·0203	·0091	—·0112	·109	·095	+·014
12735	·1134	·1452	·8291	·8604	—·0171	·0090	—·0081	·102	·092	+·010
12467	·1227	+·0290	·8925	·8746	+·0095	·0085	+·0180	·062	·083	—·021
12577	·1282	—·0401	·8925	·8830	+·0050	·0082	+·0132	·062	·077	—·015
12466	·1356	·1328	·8843	·8943	—·0053	·0078	+·0025	·067	·070	—·003
12578	·1408	·1973	·9040	·9022	+·0010	·0076	+·0086	·055	·064	—·009
12740	·1454	·2536	·8843	·9090	—·0130	·0073	—·0057	·067	·060	+·007
12579	·1532	·3469	·9107	·9204	—·0051	·0068	+·0017	·051	·053	—·002
12739	·1598	·4236	·8909	·9297	—·0203	·0064	—·0139	·063	·047	+·016
12580	·1659	·4917	·9091	·9380	—·0150	·0061	—·0089	·052	·042	+·010
12457	·1706	·5421	·9397	·9442	—·0023	·0058	+·0035	·034	·038	—·004
12738	·1730	·5673	·8875	·9473	—·0312	·0056	—·0256	·065	·036	+·029
12581	·1781	·6188	·9432	·9535	—·0053	·0053	·0000	·032	·032	·000
12737	·1857	·6909	·9277	·9623	—·0178	·0048	—·0130	·041	·026	+·015
12742	·1885	·7159	·9415	·9654	—·0122	·0046	—·0076	·033	·024	+·009
12920	·1990	·8016	·9877	·9758	+·0060	·0039	+·0099	·007	·018	—·011
12460	·2047	·8423	·9840	·9808	+·0016	·0035	+·0051	·009	·014	—·005
12919	·2115	·8852	·9949	·9860	+·0044	·0030	+·0074	·003	·011	—·008
12461	·2170	·9152	·9696	·9897	—·0101	·0026	—·0075	+·017	·009	+·008
12918	·2240	·9471	1·0438	·9936	+·0249	·0021	+·0270	—·023	·006	—·029
12462	·2300	·9686	·9519	·9962	—·0225	·0017	—·0208	+·027	·004	+·023
12917	·2365	·9856	1·0457	·9982	+·0235	·0012	+·0247	—·024	·002	—·022
12463	·2424	·9954	·9732	·9994	—·0132	—·0008	—·0124	+·015	·001	+·014
12992	·2494	1·0000	1·0228	1·0000	+·0113	·0000	+·0113	—·012	·000	—·012
12464	·2549	·9981	·9877	·9998	—·0061	+·0001	—·0062	+·007	·000	+·007
12991	·2623	·9881	1·0005	·9985	+·0010	·0006	+·0004	·000	·000	·000
12465	·2673	·9764	·9931	·9971	—·0020	·0010	—·0030	+·004	·001	+·003
12961	·2707	·9664	1·0515	·9959	+·0275	·0012	+·0263	—·027	·001	—·028
12480	·2794	·9325	·9913	·9918	—·0003	·0019	—·0022	+·005	·002	+·003
12960	·2812	·9241	1·0285	·9907	+·0188	·0020	+·0168	—·015	·003	—·018
12481	·2918	·8652	·9913	·9836	+·0038	·0027	+·0011	+·005	·006	—·001
12959	·2937	·8530	·9949	·9821	+·0064	·0029	+·0035	+·003	·007	—·004
12482	·3042	·7769	·9732	·9728	+·0002	·0036	—·0034	+·015	·011	+·004
12908	·3072	·7526	·9537	·9698	—·0082	·0038	—·0120	·026	·012	+·014
12483	·3167	·6689	·9356	·9596	—·0123	·0044	—·0167	·037	·018	+·019
12907	·3197	·6404	·9732	·9562	+·0086	·0046	+·0040	·015	·019	—·004
12754	·3262	·5756	·9519	·9483	+·0019	·0050	—·0031	·027	·023	+·004
12484	·3291	·5454	·9397	·9446	—·0025	·0052	—·0077	·034	·025	+·009
12753	·3351	·4807	·9277	·9367	—·0046	.0056	—·0102	·041	·029	+·012
12485	·3416	·4075	·9328	·9278	+·0026	·0060	—·0034	·038	·034	+·004
12752	·3478	·3351	·9660	·9190	+·0243	·0064	+·0179	·019	·039	—·020
12486	·3539	·2620	·9192	·9100	+·0048	·0067	—·0019	·046	·044	+·002

TABLE 2 *(continued)*.

plate	phase from minimum	$\cos 2\vartheta$	$l^2_{obs.}$	$l^2_{ell.}$	$l_{obs.} - l_{ell.}$	reflection	$l_{obs.} - l_{ell.+refl.}$	brightness from maximum obs.	brightness from maximum comp.	O-C
								m	m	m
12593	·3576	—·2170	·9074	·9046	+·0015	+·0069	—·0054	+·053	·047	+·006
12602	·3591	·1985	·9209	·9023	+·0097	·0070	+·0027	·045	·048	—·003
12592	·3700	·0628	·8975	·8858	+·0062	·0076	—·0014	·059	·057	+·002
12603	·3715	—·0440	·8925	·8835	+·0048	·0077	—·0029	·062	·058	+·004
12591	·3807	+·0715	·8445	·8694	—·0134	·0082	—·0216	·092	·067	+·025
12604	·3841	·1141	·8762	·8642	+·0065	·0083	—·0018	·072	·070	—·002
12590	·3912	·2021	·8570	·8535	+·0018	·0087	—·0069	·084	·076	+·008
12605	·3957	·2571	·8713	·8468	+·0132	·0089	+·0043	·075	·080	—·005
12589	·4035	·3505	·8368	·8354	+·0008	·0092	—·0084	·097	·087	+·010
12606	·4089	·4133	·8586	·8277	+·0168	·0094	+·0074	·083	·092	—·009
12456	·4115	·4428	·8958	·8241	+·0387	·0095	+·0292	·060	·094	—·034
12588	·4161	·4938	·8244	·8179	+·0036	·0097	—·0061	·105	·098	+·007
12729	·4181	·5155	·8461	·8153	+·0169	·0098	+·0071	·091	·099	—·008
12587	·4285	·6228	·8199	·8022	+·0098	·0101	—·0003	·108	·114	—·006
12730	·4306	·6432	·8154	·7997	+·0087	·0102	—·0015	·111	·117	—·006
12586	·4411	·7383	·7646	·7881	—·0133	·0105	—·0238	·146	·133	+·013
12731	·4430	·7543	·7845	·7862	—·0010	·0105	—·0115	·132	·136	—·004
12585	·4535	·8341	·7561	·7764	—·0116	·0108	—·0224	·152	·151	+·001
12732	·4554	·8470	·7465	·7749	—·0163	·0108	—·0271	·159	·154	+·005
12584	·4657	·9085	·7194	·7674	—·0278	·0110	—·0388	·179	·168	+·011
12733	·4680	·9202	·7182	·7659	—·0277	·0110	—·0387	·180	·171	+·009
12576	·4741	·9475	·7247	·7626	—·0320	·0111	—·0431	·175	·178	—·003
12572	·4762	·9556	·7302	·7616	—·0182	·0111	—·0293	·171	·180	—·009
12575	·4865	·9856	·6986	·7580	—·0348	·0112	—·0460	·195	·188	+·007
12573	·4886	·9898	·7302	·7575	—·0159	·0112	—·0271	·171	·189	—·018
12574	·4990	·9999	·7115	·7562	—·0261	+·0113	—·0374	·185	·192	—·007

However, as the areas covered during eclipses are only small, the rectified minima being shallow, no effect of any importance upon the orbit determination is to be feared.

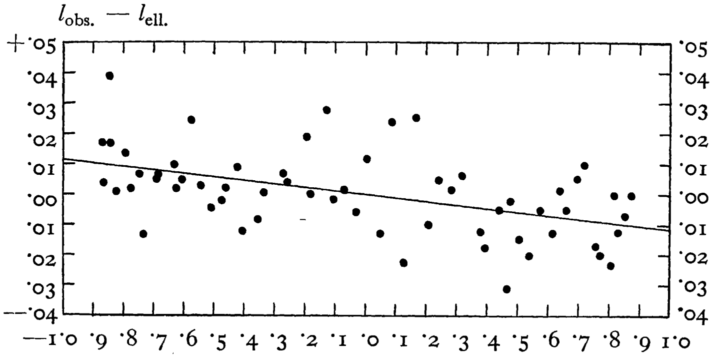

FIGURE 4.

According to KOPAL's theory[1]) of the reflection in eclipsing binaries, the amount of reflection may be expressed by the formula:

$$(b_1 - b_2) \cos \psi + \frac{20}{9\pi^2} (b_1 + b_2) \cos 2\psi,$$

ψ being the phase angle, connected to the system's anomaly ϑ by the expression:

$$\cos \psi = \sin i \cos \vartheta,$$

so that we have found:

$$(b_1 - b_2) \sin i = -·0115 \pm ·0014 \text{ (m.e.)}.$$

It is instructive to compare this value with that predicted by EDDINGTON's formula[2]). Anticipating upon the results of the determination of orbital and geometric elements of the system later in this section, this formula gives: $(b_1 - b_2) \sin i = -·0349$, about three times the amount actually found. This result is in accordance with experiences obtained by computers of other eclipsing binaries, for instance by WESSELINK[3]) in the case of SZ Cam.

As after the correction for reflection the representation of the points in Figure 3 by the straight line is improved, the mean error in the value found for $\varepsilon^2 \sin^2 i$ was recomputed; this mean error thus is reduced from $\pm$ ·0165 to $\pm$ ·0146.

The value found before for $\varepsilon^2 \sin^2 i$ is a blend of the effects from tidal elongation and the second term in the reflection, both effects depending on $\cos 2\vartheta$. The effect of this second term is to flatten the maxima, so that its effect upon the light curve is opposite to that from tidal elongation. Consequently in order to free the value found already for $\varepsilon^2 \sin^2 i$ from the effect of reflection, a positive correction is needed,

1) *Ap. J.* **89**, p. 323; see also EDDINGTON, *M.N.* **86**, p. 322.
2) *M.N.* **86**, p. 320.
3) *Thesis for the doctor's degree*, Leiden, 1938, p. 56.

the amount of which will be discussed later in this paper.

Plotting the data from Table 2, column 8, against phase, the rectified intensity curve of the variable is obtained; it is represented by Figure 5. The first

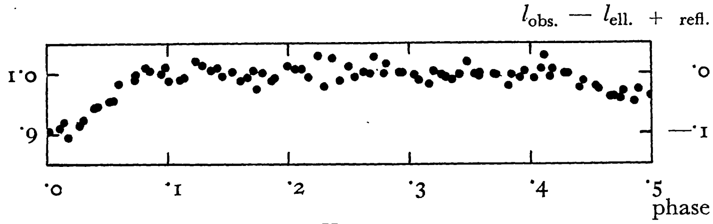

FIGURE 5.

quantity to be derived from it is the ratio of the surface brightnesses J_1/J_2. This quantity is equal to the ratio of the light lost in primary and secondary minimum of the rectified light curve respectively. It was found to be:

$J_1/J_2 = 2{\cdot}32 \pm {\cdot}17$(m.e.), equivalent to $-^{m}{\cdot}91 \pm^{m}{\cdot}08$.

As especially the secondary minimum is very shallow, the uncertainty in this determination is rather great, over 7 percent.

The data of the rectified intensity curve have now been used for the next step in the computation, viz. a determination of the ratio of the radii k, the orbital radius a in terms of the radius of the bigger component and the orbital inclination i. The least squares method for differential improvement of the orbit of an eclipsing system exposed by the author in a previous paper[1]) was followed.

Dividing the ratio of the intensities of the spectra of the two components as estimated by Miss MAURY[2]), viz. 1/·65, by the ratio of the surface brightnesses already found, we obtain the square of the ratio of the radii $k^2 = ({\cdot}81)^2$. RUDNICK and ELVEY[3]) give the value $k = {\cdot}90$. With the mean of these two values for k, and the values given by RUDNICK and ELVEY for a and i, the following set of initial values:

$$k = {\cdot}86$$
$$a = 2{\cdot}50$$
$$i = 62^\circ$$

was used as a starting point in the least squares solution. Applying the method to the principal minimum alone, we find the following corrections to the elements:

$$dk = +{\cdot}06 \pm {\cdot}49 \text{ (m.e.)}$$
$$da = +{\cdot}16 \pm {\cdot}83 \text{ (m.e.)}$$
$$di = +0^\circ{\cdot}25 \pm 6^\circ{\cdot}9 \text{ (m.e.)}$$

As was to be expected, the mean errors are even worse than those found in the case of V Pup in *B.A.N.* No. 317. This is entirely due to the fact that the eclipses of μ_1 Scorpii are much shallower. It is in the nature of things that for an eclipsing binary with shallow minima a solution for k, a and i from the photometric observations alone is not able to yield reliable values. The best policy in this case therefore seems to be to adopt the ratio of the line intensities in the spectrum as estimated by Miss MAURY as correctly representing the ratio of the luminosities of the two components. As has been already stated, the value for k derived from this datum is $k = {\cdot}81$. Now a least squares solution was made for differential corrections to the remaining two quantities a and i only. The solution was made for the two minima together, secondary minimum being given a weight of $\left(\frac{J_2}{J_1}\right)^2 = \frac{1}{(2{\cdot}32)^2}$ only with respect to principal minimum. The result is:

$k = {\cdot}81$ (adopted from Miss MAURY's estimates of line intensity ratios)

$a = 2{\cdot}486 \pm {\cdot}057$ (m.e.)

$i = 61^\circ{\cdot}10 \pm 0^\circ{\cdot}86$ (m.e.).

The improvement in the mean errors in a and i, now that a fixed value for k has been adopted, is conspicuous. Therefore the initiative of PETRIE[4]) for measuring the line profiles in spectrograms of eclipsing binaries and deriving the light ratio of the two components directly from these in stead of from the light curve is much to be welcomed.

In Table 2, column 10, the brightness of the variable has been given as computed from the final set of elements. Column 9 shows the observed brightness, column 11 the difference observed minus computed. The mean error of a single plate with respect to this computed lightcurve (drawn as a full line in Figure 2) is $\pm^{m}{\cdot}0125$. Disregarding systematic errors like night errors the total weight of the light curve therefore is $81/({\cdot}0125)^2$ = about 500 000 m^{-2}.

As has been remarked by several authors[5]) the value found for $z = \varepsilon^2 \sin^2 i$ by the usual procedure is affected by reflection. The correction necessary to free $\varepsilon^2 \sin^2 i$ from this effect is, according to KOPAL[6]), given by the expression

$$\frac{40}{9\pi^2}(b_1 + b_2) \sin^2 i.$$

As $\frac{b_1}{b_2} = \frac{J_1}{J_2} = 2{\cdot}32$, the amount of this correction can easily be computed from the value for $(b_1 - b_2) \sin i$ already found; the result is $+{\cdot}0114$.

Consequently we obtain the following value for the ellipticity constant:

$$\varepsilon^2 \sin^2 i_{\text{ corrected}} = {\cdot}2552 \pm {\cdot}0147 \text{ (m.e.)}.$$

1) *B.A.N.* No. 217, p. 328.
2) L.c.
3) L.c.
4) *Publ. Amer. Astr. Soc.* **9**, p. 162.
5) For instance by WALTER, *Königsberg Veröff.* No. 2, p. 9.
6) *Ap. J.* **89**, p. 323.

corresponding to a ratio of axes in the equatorial section:

$$\sqrt{1-\varepsilon^2} = {\cdot}8167 \pm {\cdot}0118 \text{ (m.e.)},$$

very nearly the same value as found in *B.A.N.* No. 317 for the system V Puppis, which in many respects (period, spectral type, relative radial velocity of components) is so strikingly similar to μ_1 Scorpii.

As in the system μ_1 Scorpii the two components are rather unequal the ratio of axes in their equatorial sections can also be expected to be unequal. The tidal elongation found from the light curve therefore is only a mean value for the elongations of the two components[1]). It is instructive to compare the value found with the theoretical ones derived from sizes, masses and mutual distance of the components by CHANDRASEKHAR's formulae[2]). The weighted mean for the two components, luminosity being taken as weight, was derived for the MacLaurin model (uniform density) and the Roche model (all mass concentrated in the centre) respectively. The values found are:

$$\sqrt{1-\varepsilon^2}_{\text{MacLaurin}} = {\cdot}798$$
$$\sqrt{1-\varepsilon^2}_{\text{Roche}} = {\cdot}929$$

It is gratifying to see that the value derived from the observations is lying within these limits, although the homogeneous model seems to be favoured, a conclusion reached also by other investigators for similar systems[3]). By the aid of RUSSELL's table[4]) the following value was found for the ratio between central density and mean density:

$$\frac{\rho_c}{\rho_m} = 3{\cdot}37.$$

Although the uniform solution must be expected to be much nearer to the truth than a fully darkened one, the axial ratio of the equatorial section was also determined from the light curve under the latter supposition; its value was found to be:

$$\sqrt{1-\varepsilon^2} = {\cdot}889,$$

still between the limits mentioned before, but now favouring a higher concentration towards the centre of the star's density.

5. *Dynamical parallax and absolute dimensions.*

A dynamical parallax was computed from the light curve and the average spectral type of the two components by the same method as has been applied by the author to the case of V Puppis[5]). Neglecting absorption of light in space and assuming the spectral types to be B1 and B6, in accordance with the difference in surface brightness found and with the mean spectral type B3 as assigned to the system by Miss MAURY, we find for μ_1 Scorpii, if we take the surface brightnesses of its components as $-3^{\rm m}{\cdot}53$ and $-2^{\rm m}{\cdot}85$ brighter than the sun respectively, and the apparent brightness of μ_2 Scorpii as $3^{\rm m}{\cdot}68$ I Pv:

$$p = {}''{\cdot}0070,$$

corresponding to:

$$a = {\cdot}0674 \text{ astronomical units}$$

mass brighter component =	$12{\cdot}20$ ⊙
mass fainter component =	$7{\cdot}29$ ⊙
$M_{\rm vis}$ brighter component =	$-2^{\rm M}{\cdot}18$
$M_{\rm vis}$ fainter component =	$-1^{\rm M}{\cdot}72$.

More reliable values for orbital radius and masses can be derived from the spectroscopic data; they are:

$$a = {\cdot}0729 \text{ astronomical units}$$

sum of masses $m_1 + m_2 = 25{\cdot}45$ ⊙

Using Eddington's mass-luminosity relation to distribute this total mass over the two components, and to find the corresponding absolute brightnesses, the following further values result:

mass brighter component =	$16{\cdot}12$ ⊙
mass fainter component =	$9{\cdot}33$ ⊙
$M_{\rm vis}$ brighter component =	$-2^{\rm M}{\cdot}75$
$M_{\rm vis}$ fainter component =	$-2^{\rm M}{\cdot}29$

The dynamical parallax derived above compares very favorably with the group parallax found for μ_1 and μ_2 Scorpii by KAPTEYN[6]), viz. $''{\cdot}0074$ and $''{\cdot}0082$ respectively. As it should be, on account of the neglect of absorption of light in space, the dynamical parallax is smaller than the group parallax.

The spectroscopic parallaxes given by SCHLESINGER[7]) for μ_1 and μ_2 Scorpii are $''{\cdot}011$ and $''{\cdot}007$ respectively. After applying the correction for duplicity[8]), where necessary, to the sources from which SCHLESINGER's value for the spectroscopic parallax of μ_1 Scorpii has been derived, this latter one is decreased to $''{\cdot}008$, in as good agreement with the dynamical and the group parallax as could be expected.

Using KAPTEYN's parallax as the most reliable one of the three, the following absolute magnitudes are found, no absorption being taken into account:

$M_{\rm vis}$ brighter component =	$-2^{\rm M}{\cdot}06$
$M_{\rm vis}$ fainter component =	$-1^{\rm M}{\cdot}60$

The surface brightnesses corresponding to these figures are $-3^{\rm m}{\cdot}69$ and $-2^{\rm m}{\cdot}77$ respectively with respect to the sun, or 29·9 and 12·8 times the sun's

1) Cf. LUYTEN, *M.N.* **98**, p. 464.
2) *M.N.* **93**, pp. 548, 551.
3) Cf. WALTER, *Königsberg Veröff.* No. 2, p. 21; KOPAL, *M.N.* **96**. p. 856.
4) *M.N.* **88**, p. 643.
5) *B.A.N.* No. 317, p. 330.
6) *Ap. J.* **40**, p. 120.
7) SCHLESINGER, *General Catalogue of Stellar Parallaxes.*
8) See EDWARDS, *M.N.* **88**, p. 695.

surface luminosity. According to HERTZSPRUNG's formula[1]) these surface brightnesses correspond to temperatures of 20 000° and 12 500° respectively[2]). If there is already absorption to be taken into account at the relatively short distance of 140 parsecs for μ_1 Scorpii, these temperatures would have to be raised.

In the following list the principal data now obtained are assembled:

Epoch of principal minimum	J.D. 2428414^d·2978 $\pm$ ^{d}·0022
Period	1^d·4462683 $\pm$ ^{d}·0000004
Brightness at maximum, μ_2 Sco = 3^m·68,	3^m·05
Brightness at principal minimum	3^m·35
Brightness at secondary minimum	3^m·24
Ellipticity constant $\varepsilon^2 \sin^2 i$, corrected for reflection	·2552 $\pm$ ·0147
Reflection constant $(b_1 - b_2) \sin i$	·0115 $\pm$ ·0014
Ratio of surface brightnesses J_1/J_2	2·32 $\pm$ ·17
Light of brighter component, in fraction of total light	·606
Ratio of radii k	·81
Orbital radius a, in terms of radius of bigger component	2·486 $\pm$ ·057
Inclination of orbit i	61°·10 $\pm$ °·86
Oblateness of equatorial section $\sqrt{1-\varepsilon^2}$	·8167
Orbital radius a in km	10 890 000 km = 15·66 r$_\odot$ = ·0729 astr. units (=″·00054 for π=″·0074)
Longest radius of brighter component	3 548 000 km = 5·10 r$_\odot$
Longest radius of fainter component	4 381 000 km = 6·30 r$_\odot$
Mass of brighter component	16·12 $\odot$
Mass of fainter component	9·33 $\odot$
Density of brighter component	·223 $\rho_\odot$
Density of fainter component	·068 $\rho_\odot$
Mean absolute brightness of brighter component	−2^M·06
Mean absolute brightness of fainter component	−1^M·60
Surface brightness of brighter component	−3^m·69 = 29·9 × $\odot$
Surface brightness of fainter component	−2^m·77 = 12·8 × $\odot$
Effective temperature of brighter component	20 000°
Effective temperature of fainter component	12 500°
Dynamical parallax	″·0070
Group parallax (KAPTEYN)	″·0074
Spectroscopic parallax (SCHLESINGER, corrected for duplicity)	″·008

A drawing of the system is given in Figure 6. On the scale of this drawing the star μ_2 Scorpii would be at a distance of 18 km, thus illustrating the extremely unequal ratio of the distances between the components of the visual pair on one hand and between the components of the eclipsing pair on the other hand. If to μ_2 Scorpii the same mass is assigned as has been found for the combined mass of the system μ_1 Scorpii, the period of the visual pair must be at least 1 400 000 years.

I am indebted to Mr. C. J. KOOREMAN for measuring and reducing the plates, and to Mr. E. W. DE ROOY for preparing the drawings for the diagrams in this paper.

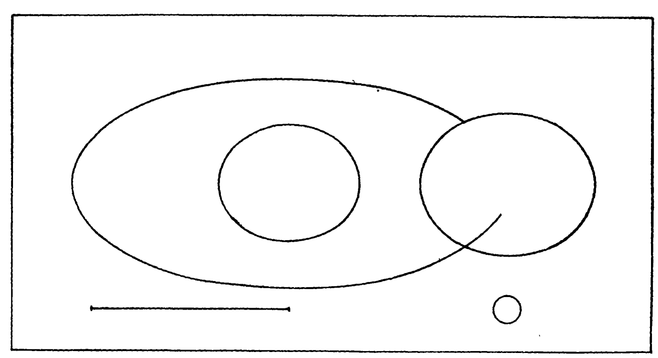

FIGURE 6.

1) *Zeitschrift für wissenschaftliche Photographie*, **4**, p. 43. See also EDDINGTON, *The internal constitution of the stars*, p. 139.
2) Compare also KUIPER, *Ap. J.* **88**, p. 443.