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COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

Photographic observations of RR Lyrae from plates taken at Potsdam 1910—1913, by Ejnar Hertzsprung.

The variable star RR Lyrae is the brightest known of the so called cluster type and therefore specially fit for detailed investigation. In fact, it is the only star of this kind for which spectroscopic elements have been determined (C. C. KIESS, *Lick Bull.* 232, 7, 140; 1913).

In addition to the variables of the δ Cephei type mentioned in *B. A. N.* 13, RR Lyrae was observed with the UV Zeiss triplet ($a = 15$ cm., $f = 150$ cm.) at Potsdam. The plates (20 cm. by 20 cm. or 16 cm. by 16 cm.) were taken 3 mm. intrafocal. The diameter of the star images is therefore 3 mm. The coordinates of the variable v and the comparison stars are given in Table 1.

TABLE 1.

Star	B.D.	α (1855)			δ (1855)	UV mag.
		h	m	s		
a	+42°3325	19	18	23.5	+42 41.2	6.686
v	42°3338	20	50.5		42 30.8	(7.07—8.19)
f	42°3340	21	23.4		42 2.1	7.978
m	41°3350	22	14.8		41 10.9	—
c	41°3352	22	42.3		41 56.0	7.186
g	41°3354	22	55.6		41 21.1	8.224
l	42°3357	24	24.1		42 52.7	—
h	42°3359	24	38.0		42 20.9	8.251
e	42°3364	25	51.5		42 51.6	8.022
x	42°3372	26	38.5		42 39.9	8.244

The UV photographic magnitudes given in Table 1*) are only apparent, i. e. not corrected for position on the plate or differential extinction and referred to an arbitrary zero. The fainter comparison stars of similar apparent magnitude were finally combined into pairs

*) For the variable the mean range of variation is given.

or groups as follows: ef , ghx and lm with mean apparent magnitudes of 8.000, 8.240 and 8.668 respectively.

The comparison star a and the pair lm were used in a few exceptional cases only.

All the plates here considered were taken between 18^h59^m and 0^h31^m sidereal time and no correction for change in differential extinction was needed. The telescope was used in the position to the east of the pier.

From 1910 Sept. 2 to 1913 Oct. 21 I obtained 155 plates on 39 nights with 1010 exposures altogether. The usual exposure time was 3 minutes. An account of these plates is given in Table 2.

Moreover 6 plates were taken with a grating placed in front of the objective (*A. N.* 4916) in order to determine the apparent differences in magnitude between the comparison stars.

The plates have been measured in the Hartmann microphotometer by Frl. H. MATTENKLODT. A few revisions especially in the ascending part of the light curve have been made here in Leiden with the aid of a thermoelectric arrangement constructed with the kind help of Prof. F. ZERNIKE (*cf.* J. SCHILT, *B. A. N.* 10).

The details of each separate exposure are given in Table 3 (pp. 144—147). The $J.D.$ is given in heliocentric Greenwich M. T. The last exposure of each plate is marked by a point after the fraction of the $J.D.$ The phase P has been calculated according to the formula

$$P = 1.7641511 (J.D. \text{ hel. M. T. Grw.} - 2418791).$$

corresponding to a period of 0.56684488. This period was derived from my own observations. As pointed out by R. PRAGER (*Sitzungsber. d. Kön. preuss. Akad. d. Wiss.* 8, 216; 1916) the period is subjected to slow secular change. It has not yet been decided, whether this change is of periodic character or not.

TABLE 2.

Date	Nr. of first plate	Sid. time Potsdam middle		Number of		J. D.	Phase observed		Correction to phase for 40d.6 period
		first exp.	last exp.	plates	exp.		first exp.	last exp.	
		h m	h m				P	P	P
1910 Sept. 2	330	22 19	0 15	4	33	2418917	223'073	'215	+ '023
4	336	21 49	0 24	5	41	8919	226'556	'744	+ '024
5	344	21 59	0 15	5	40	8920	228'327	'493	+ '023
10	335	21 58	0 1	5	34	8925	237'122	'273	+ '013
11	362	22 25	23 45	3	24	8926	238'914	'012	+ '011
12	368	23 22	23 27	1	3	8927	240'743	'749	+ '008
26	408	22 32	22 54	1	8	8941	265'312	'339	— '023
29	421	21 59	23 46	4	32	8944	270'550	'680	— '018
Oct. 1	432	21 52	22 37	2	17	8946	274'059	'115	— '011
7	462	22 3	22 26	1	9	8952	284'629	'657	+ '009
8	474	21 45	22 34	2	17	8953	286'366	'425	+ '013
13	483	22 29	23 22	3	9	8958	295'216	'281	+ '023
25	531	21 49	22 3	1	6	8970	316'278	'295	— '001
27	535	21 44	22 25	2	16	8972	319'791	'840	— '007
Nov. 5	545	22 16	22 32	1	7	8981	335'663	'682	— '024
17	594	21 52	22 43	6	12	8993	356'744	'807	+ '010
1911 Aug. 24	1261	22 6	0 31	5	39	9273	851'140	'317	— '004
26	1273	22 24	23 57	3	25	9275	854'681	'794	+ '003
27	1282	21 44	0 24	6	50	9276	856'392	'586	+ '007
31	1305	22 12	23 17	3	22	9280	863'462	'542	+ '019
Sept. 3	1342	22 41	0 12	3	27	9283	868'776	'887	+ '023
20	1420	21 48	23 50	4	33	9300	898'620	'769	— '018
Oct. 16	1459	23 6	0 31	3	22	9326	944'457	'500	+ '023
1913 Aug. 22	2604	21 33	22 9	2	12	2420002	2137'172	'217	— '010
25	2643	19 24	22 26	8	48	0005	2142'294	'515	— '000
26	2667	20 23	22 9	4	24	0006	2144'125	'254	+ '004
27	2690	20 14	22 22	6	36	0007	2145'873	'030	+ '006
28	2712	19 25	22 26	8	48	0008	2147'572	'794	+ '010
Sept. 1	2756	18 59	20 47	4	20	0012	2154'577	'688	+ '020
6	2760	19 13	21 21	5	28	0017	2163'393	'548	+ '023
7	2771	20 38	21 58	3	18	0018	2165'255	'352	+ '022
10	2837	19 32	22 54	8	49	0021	2170'452	'698	+ '015
12	2865	20 38	22 1	4	24	0023	2174'051	'153	+ '008
23	2969	21 14	22 11	3	14	0034	2193'448	'517	— '023
26	3008	20 55	21 57	3	18	0037	2198'701	'778	— '023
27	3056	21 0	0 26	8	49	0038	2200'467	'719	— '022
29	3144	21 9	23 7	4	24	0040	2203'996	'141	— '018
Oct. 1	3221	20 13	23 26	7	42	0042	2207'447	'683	— '013
21	3439	21 37	23 54	5	30	0062	2242'735	'901	+ '015

So far the differences from a uniform period are not greater than might reasonably be explained from orbital motion of RR Lyrae round another star.

In *Contrib. Mount Wilson Obs.* 112, *Ap. J.* 43, 217; 1916 HARLOW SHAPLEY has discussed the observations of RR Lyrae using as time of reference the moment, when the star passes the median magnitude on the ascending branch of the light curve. Besides the slow perturbation also announced by PRAGER, SHAPLEY found another term in the residuals $O-C$ having a period of 40 days and an amplitude of $^{\circ}026$, at least as far as the first condensed series of observations of WENDELL is concerned. SHAPLEY writes: „There can be no doubt that a real irregularity is present. An attempt was made to find a uniform period for the variation that would satisfy

all the observations. This failed in part, perhaps because of insufficient data, but it seems that for the whole series the oscillation is roughly periodic with a varying amplitude.” The present observations give further evidence of the existence of this secondary oscillation.

In the upper part of the accompanying figure (p. 143) 9 diagrams are given showing my separate observations on 14 *) different nights, when the variable was between the phases .4 and .8, which interval covers the ascending branch of the light curve and the adjacent minimum and maximum. For comparison the mean light curve, using the uniform period mentioned above,

*) The circles on the first of these 9 diagrams represent additional observations discussed in the Appendix.

has been drawn on each diagram. It is evident from these diagrams that a real irregularity is present*). In *Ap. J.* 43, 228, Table V, the observations used by SHAPLEY are given. After correction for the slow secular perturbation the residuals $O-C_2$ were formed. These residuals should therefore show the term of about 40 days period. Adding my own observations and tabulating separately the epochs E of the residuals $O-C$ exceeding ± 0.0075 , I find the list of 26 epochs given in Table 4. From this list only the epoch 8473 ($O-C_2 = +0.010$) of TOWNLEY has been excluded for discordance.

TABLE 4.

Difference $O-C$ disregarding secondary oscillation		observed epoch of great $O-C$ E_0	adopted epoch of secondary oscillation E'	calculated epoch of max. $O-C$ E_C	Diff. $E_0 - E_C$
SHAPLEY $O-C_2$	HZG. $O-C$				
d	d				
+012	—	100	0	112	-12
+10	—	114	0	112	+2
+18	—	121	0	112	+9
-9	—	144	5	148	-4
+10	—	195	1	184	+11
-11	—	218	1.5	219	-1
-12	—	225	1.5	219	+6
-10	—	285	2.5	291	-6
-8	—	578	6.5	578	0
-10	—	936	11.5	936	0
+12	—	2755	37	2762	-7
-8	—	2799	37.5	2798	+1
+18	—	5761	79	5771	-10
—	-021	7167	98.5	7168	-1
—	+10	7211	99	7204	+7
—	+11	7839	108	7848	-9
+29	—	8431	116	8421	+10
-10	—	8540	117.5	8529	+11
—	-12	9095	125.5	9102	-7
—	+14	9104	125.5	9102	+2
—	-9	9111	125.5	9102	+9
—	+14	9141	126	9138	+3
—	+9	9148	126	9138	+10
+24	—	9210	127	9209	+1
-22	—	9238	127.5	9245	-7
-9	—	9873	136.5	9890	-17

A least square solution then gave the following elements of SHAPLEY's term:

*) The 6 observations of plate no. 3056 shown on the 8th diagram between the phases .46 and .50 give an unusual faint brightness for the variable. Probably this plate has been unsuccessful. It has however not been rejected. The point marked by a circle on the same diagram represents an additional plate taken with the grating before the objective.

$$\text{Max. } (O-C) \text{ at } E-4983 = 71.634 (E-68) \pm 1.6 \pm 0.28 \quad (\text{m. e.})$$

$$\text{or at } J.D. 2417681^{\text{d}}.2 + 40^{\text{d}}.605 (E-68) \pm .9 \pm 0.16 \quad (\text{m. e.})$$

The difference $E_0 - E_C$ exceeds one quarter of the period (71.634), only in the case of the excluded observation at $E=8473$, the mean square of the tabulated residuals $E_0 - E_C$ being $63.5 = (\pm 8.0)^2$.

From my own observations alone I derived for the 12 epochs given in Table 5 the phase $P_{7.6}$ at which

TABLE 5

$E-6941$	$\frac{E}{71.637}$	θ	$P_{7.6}$ Phase of mag 7.6 ascending branch		
			O	C	$O-C$
	P'	°	P	P	P
226	100.04	14	.535	.550	-.015
270	100.66	238	.589	.591	-.002
856	108.84	302	.566	.566	-.000
863	108.94	338	.567	.554	+.013
898	109.42	151	.592	.591	+.001
944	110.07	25	.564	.550	+.014
2147	126.86	310	.559	.564	-.005
2154	126.96	346	.550	.553	-.003
2163	127.08	29	.548	.550	-.002
2170	127.18	65	.556	.558	-.002
2200	127.60	216	.596	.596	+.000
2207	127.70	252	.588	.586	+.002

the magnitude 7^m.6 was reached on the ascending branch of the light curve. The phases P' of the 40^d.6 oscillation were counted from SHAPLEY's $E=O$ (corresponding to -6941 in my counting) as zero point. A least square solution then gave the following formula

$$P_{7.6} = P.5735 - P.0062 \sin \theta - P.0229 \cos \theta$$

where θ is the phase P' expressed in angle.

The corresponding range of the 40^d.6 oscillation is $\pm P.0237 \pm P.0036$ (m. e.) or $\pm 0.0134 \pm 0.0020$ (m. e.).

This is the same as found by SHAPLEY from WENDELL's observations (*Ap. J.* 43, 231). Having regard to the period of 40^d.6 this range appears much too great to be reasonably explained by orbital motion of RR Lyrae round another star, which would give changes in radial velocity of the order of 10³ km/sec. It is the first variable star for which such a secondary oscillation has been indicated. This most interesting result of SHAPLEY is confirmed as far as the present observations go, but more material is required to settle the question definitively.

As to the varying height of the maximum, the observations are not sufficient to decide whether this variation has the same period of 40.6 days. If so, the evidence there is tends to show that the maximum is brightest when the period is shortest. This would imply that the „mean” light curve is never presented by the variable because when the time of passing the median magnitude is normal the maximum will either be higher or lower than in the mean.

Nevertheless I have computed a mean light curve. For this purpose the effect of the 40.6 oscillation must first be eliminated. Therefore corrections to the phases P (as given in Table 3) were calculated according to the formula for $P_{7.6}$ given above. The corrections thus used for each separate night are contained in the last column of Table 2. Dividing the 1010 observations into groups of 40 or 30, I find the values of Table 6 which are shown graphically on the

TABLE 6.

number of exposures	mean corrected phase	mean magni- tude	number of exposures	mean corrected phase	mean magni- tude
	P	m		P	m
40	.0006	7.911	30	.5900	7.361
40	.0737	8.048	30	.6092	7.208
40	.1256	8.091	30	.6279	7.118
40	.1720	8.125	30	.6433	7.087
40	.2106	8.123	30	.6567	7.066
40	.2571	8.113	30	.6768	7.127
40	.3030	8.135	30	.6959	7.198
40	.3678	8.118	40	.7192	7.264
40	.4160	8.111	40	.7537	7.410
40	.4451	8.165	40	.7924	7.514
40	.4728	8.194	40	.8416	7.598
40	.4942	8.170	40	.9205	7.784
30	.5116	8.136			
30	.5293	8.074	37	.4447	8.148
30	.5501	7.891	37	.4734	8.175
30	.5710	7.670			

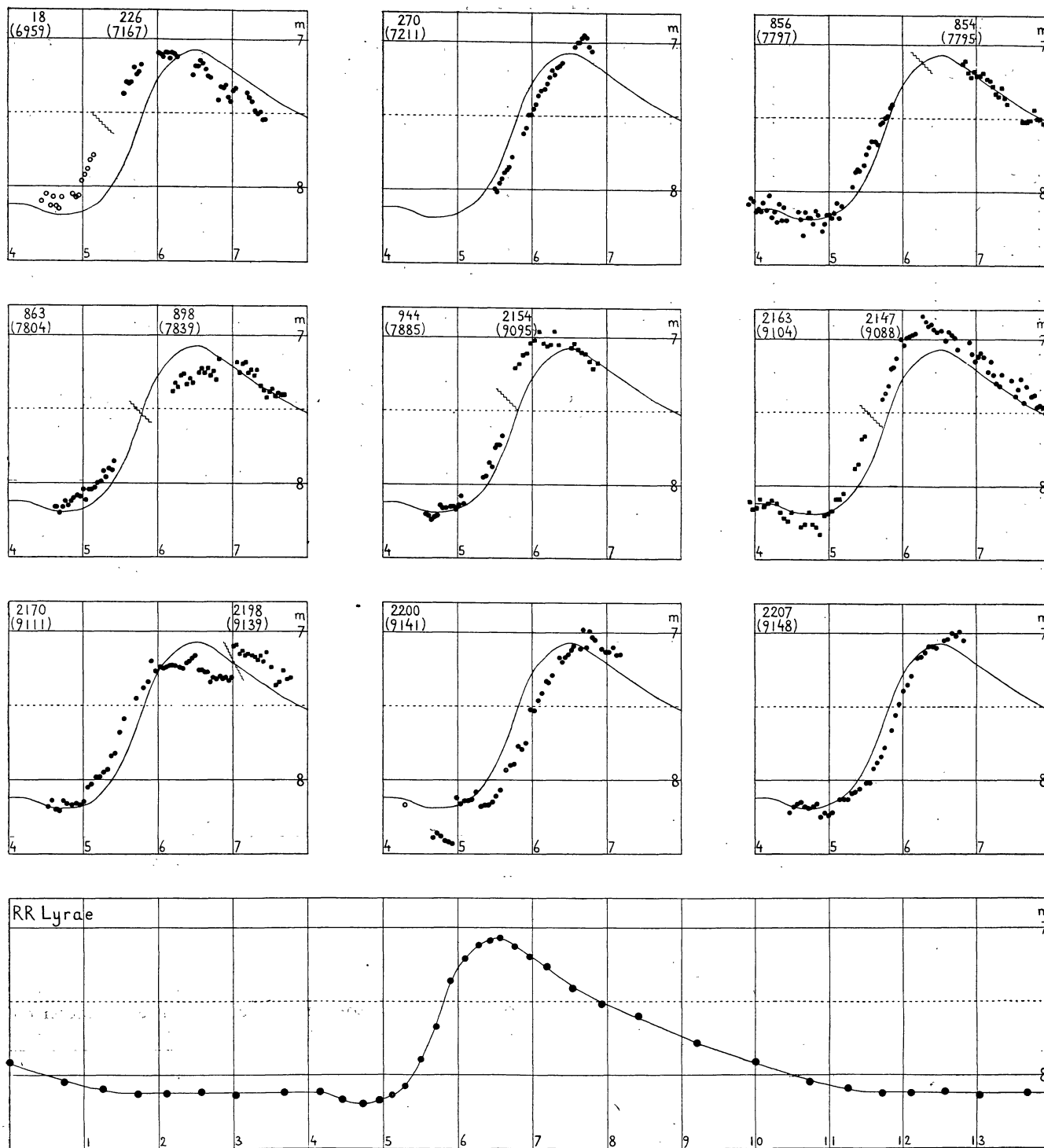
diagram at the bottom of the accompanying figure *). This light curve shows the usual particulars of stars of this kind. The quickest rise in brightness on the ascending branch is very nearly at the rate of 1

*) This diagram is directly comparable with those given in *B. A. N.* 13 of X , V , Z and RR Lacartae and in *A. N.* 4916 of S Sagittae.

magnitude per hour. The mean UV photographic range is $8^m.19 - 7^m.07 = 1^m.12$, while from WENDELL's visual observations $7^m.67 - 6^m.82 = 0^m.85$ is found. The difference $1^m.12 - 0^m.85 = 0^m.27$ gives the corresponding range in colour-index. Between the phases .15 and .42, or during about one quarter of the period, the brightness of the variable does not change sensibly. Between the phases .42 and .47 the brightness decreases by $8^m.19 - 8^m.12 = 0^m.07$ and then rises to maximum. This detail is only little changed (min. $8^m.17$) when the plate No. 3056 already mentioned is left out for discordance. The tenth and eleventh normal places of Table 5 would then be altered as shown at the end of that table below the line. It is remarkable that the same peculiarity has already been announced for variables in the cluster Messier 3 by J. LARINK (*Astron. Abh. d. Hamburger Sternw. in Bergedorf*, 2, Nr. 6, 6; 1922). In a particularly striking way this feature is shown in the photographic light curve of SS Cancri by W. BAADE (*Mitt. d. Hamburger Sternw. in Bergedorf*, 5, Nr. 14, 23; 1922). This peculiarity therefore seems to be characteristic for variables of this type.

The accuracy of the present observations has been evaluated in the following way. The mean square of the difference in magnitude between the last exposure of one plate and the first on the next following plate is, taking account of the variability of the star, found to be $2 \times .00311 m^2$, while the mean square of the difference between two consecutive exposures on the same plate is $2 \times .00138 m^2$. Hence the square of the systematic mean error of one plate is $.00311 - .00138 = .00173 m^2$. Accordingly the weight of a plate carrying n exposures is $1/((.00173 + .00138/n) m^2)$. For $n=6$ the weight is $510 m^{-2}$. The total weight of all the 155 plates calculated in this way is $78750 m^{-2}$. If a new plate had been used for each exposure the total weight would have been $1010/.00311 = 325000 m^{-2}$ or about 4 times as large. But besides the increased expense and work the number of exposures per hour would have been considerably less in that case.

It is seen from the foregoing that many observations are still needed to unveil the true character of the perturbations in the light curve of RR Lyrae. The star may therefore be recommended to further attention. Owing to the short period, collaboration between observatories at different longitude, may be useful. Photographic observations are preferable because of the danger of bias in visual work in the case of a quickly changing star and because the range is greater photographically than visually.



Upper diagrams: Individual observations of RR Lyrae on 15 different nights, compared with mean light curve.

Lower diagram: Mean light curve of RR Lyrae.

TABLE 3.

<i>J. D. 2418917</i>			d	P	m	d	P	m	d	P	m
Hel.			5063.	705	7'33	4535	196	8'05	3821	589	7'62
M. T.			5152	720	7'36	4554	200	8'07	3840	593	7'58
Grw.			5171	724	7'39	4572	203	8'09	3859	596	7'49
d	P	m	5190	727	7'42	4591	206	8'06	3877	599	7'49
4478	223'073	8'08	5210	730	7'48	4609	209	8'08	3896	602	7'45
4498	077	8'09	5229	734	7'50	4627	213	8'10	3914	606	7'42
4518	080	8'09	5250	738	7'49	4646	216	8'00	3933	609	7'36
4539	084	8'08	5269	741	7'54	4760	236	8'07	3952	612	7'33
4560	088	8'11	5287.	744	7'54	4779	239	8'06	3982	618	7'32
4579	091	8'11	<i>J. D. 2418920</i>			4798	243	7'99	4000	621	7'28
4598	094	8'13	d	P	m	4816	246	7'98	4019	624	7'24
4617.	098	8'12	4260	228'327	8'08	4835	249	7'96	4038	627	7'19
4647	103	8'08	4280	330	8'12	4854	252	8'10	4056	631	7'22
4666	106	8'06	4298	334	8'09	4872	256	7'94	4074	634	7'17
4685	110	8'08	4317	337	8'13	4890.	259	8'02	4097	638	7'16
4704	113	8'11	4337	341	8'11	4917	264	8'12	4115.	641	7'14
4723	116	8'11	4356	344	8'13	4969.	273	8'10	4210	658	7'03
4742	120	8'16	4374	347	8'12	<i>J. D. 2418926</i>			4228	661	7'00
4761	123	8'16	4394.	351	8'11	d	P	m	4247	664	7'00
4780.	126	8'17	4425	356	8'10	4271	238'914	7'88	4265	668	6'97
4925	152	8'17	4444	359	8'11	4294	918	7'88	4284	671	6'95
4944	155	8'21	4463	363	8'10	4313	921	7'91	4302	674	6'97
4964	159	8'28	4481	366	8'08	4332	925	7'93	4320	677	7'03
4983	162	8'15	4500	369	8'14	4350	928	7'94	4339.	680	7'06
5002	166	8'20	4519	373	8'08	4369	931	7'92	<i>J. D. 2418946</i>		
5022	169	8'27	4538	376	8'11	4387	934	7'93	d	P	m
5033	171	8'20	4556.	379	8'11	4406.	938	7'94	3491	274'059	8'03
5052	174	8'26	4668	399	8'04	4516	957	7'88	3509	062	8'02
5071.	178	8'15	4686	402	8'11	4537	961	7'95	3527	066	8'04
5142	190	8'12	4705	406	8'13	4557	964	7'88	3546	069	8'02
5162	194	8'17	4724	409	8'14	4577	968	7'88	3571	073	7'99
5182	197	8'14	4742	412	8'13	4596	971	7'92	3582	075	7'96
5201	200	8'21	4761	415	8'16	4616	975	7'93	3600	079	8'05
5220	204	8'20	4779	419	8'16	4635	978	7'96	3619.	082	8'07
5242	208	8'17	4798.	422	8'15	4654.	982	8'00	3656	088	8'05
5261	211	8'14	4828	427	8'11	4690	988	7'95	3674	092	8'07
5281.	215	8'15	4846	430	8'09	4711	992	7'96	3692	095	8'05
<i>J. D. 2418919</i>			4865	434	8'10	4730	995	7'92	3710	098	8'10
d	P	m	4884	437	8'06	4749	998	7'92	3728	101	8'08
4218	226'556	7'37	4902	440	8'14	4770	239'002	7'92	3751	105	8'13
4237	559	7'29	4918	443	8'12	4789	005	7'91	3770	108	8'17
4255	562	7'30	4941	447	8'13	4809	009	7'92	3788	112	8'15
4274	565	7'29	4960.	450	8'13	4829.	012	8'00	3806.	115	8'11
4293	569	7'19	5070	470	8'11	<i>J. D. 2418927</i>			<i>J. D. 2418952</i>		
4312	572	7'24	5089	473	8'09	d	P	m	d	P	m
4331	575	7'22	5108	477	8'07	4637	240'743	7'58	3403	284'629	7'09
4349.	579	7'17	5126	480	8'07	4655	746	7'63	3421	632	7'08
4476	601	7'09	5145	483	8'09	4673.	749	7'63	3440	635	7'06
4495	604	7'10	5164	486	8'06	<i>J. D. 2418941</i>			3458	638	7'06
4513	608	7'12	5184	490	8'05	d	P	m	3476	642	7'05
4533	611	7'09	5203.	493	8'00	3906	265'312	8'13	3495	645	7'03
4548	614	7'09	<i>J. D. 2418925</i>			3927	316	8'11	3520	649	7'08
4563	616	7'13	d	P	m	3958	321	8'09	3538	652	7'06
4582	620	7'09	4115	237'122	8'05	3978	324	8'14	3563.	657	7'02
4601	623	7'10	4134	126	8'01	3996	328	8'17	<i>J. D. 2418953</i>		
4619.	626	7'12	4152	129	8'03	4015	331	8'18	d	P	m
4741	648	7'24	4172	132	8'04	4034	334	8'18	3249	286'366	8'22
4760	651	7'18	4192	136	8'03	4061.	339	8'16	3268	369	8'20
4779	654	7'18	4210	139	8'04	<i>J. D. 2418944</i>			3286	372	8'13
4798	658	7'14	4228	142	8'04	d	P	m	3304	375	8'13
4817	661	7'16	4247.	146	8'02	3598	270'550	8'00	3326	379	8'10
4835	664	7'20	4263	166	8'06	3616	553	8'02	3344	382	8'12
4854	668	7'25	4381	169	8'07	3636	556	7'96	3364	386	8'12
4873.	671	7'26	4400	172	8'02	3654	560	7'93	3383.	389	8'09
4934	682	7'41	4418	176	8'02	3672	563	7'89	3419	396	8'10
4954	685	7'32	4436	179	8'04	3691	566	7'87	3438	399	8'11
4971	688	7'33	4454	182	8'01	3710	570	7'85	3456	402	8'09
4989	692	7'31	4472	185	8'05	3734.	574	7'78	3477	406	8'03
5008	695	7'39	4491.	188	8'05				3496	409	8'08
5026	698	7'42	4517	193	8'03						
5045	701	7'35									

d.	P	m	d.	P	m	d.	P	m	d.	P	m
'3512	'412	8:05	'4680	'146	8:17	'4526	'412	8:08	'4648	'782	7:62
'3549	'418	8:04	'4702	'150	8:11	'4551.	'416	8:13	'4666	'786	7:54
'3568	'422	8:06	'4723	'154	8:15	'4572	'420	8:03	'4686	'789	7:50
'3586.	'425	8:07	'4745	'158	8:16	'4590	'423	8:18	'4708	'793	7:57
<i>J. D. 2418958</i>			'4767	'162	8:09	'4609	'426	8:14	'4730	'797	7:62
d.	P	m	'4789.	'166	8:10	'4627	'430	8:21	'4751	'801	7:52
'3418	295'216	8:19	'4885	'183	8:12	'4646	'433	8:09	'4773.	'804	7:69
'3440	'220	8:16	'4907	'187	8:11	'4664	'436	8:20	'4805	'810	7:48
'3462.	'224	8:12	'4930	'191	8:10	'4682	'439	8:11	'4826	'814	7:55
'3576	'244	8:08	'4954	'195	8:10	'4701.	'443	8:20	'4848	'818	7:52
'3596	'248	8:10	'4976	'199	8:16	'4792	'459	8:14	'4870	'822	7:55
'3616.	'251	8:11	'4998	'202	8:20	'4811	'462	8:19	'4892	'825	7:55
'3734	'272	8:10	'5020	'206	8:11	'4829	'465	8:30	'4914	'829	7:60
'3755	'276	8:09	'5042.	'210	8:16	'4848	'468	8:14	'4936	'833	7:56
'3784.	'281	8:13	'5074	'216	8:10	'4866	'472	8:18	'4957	'837	7:50
<i>J. D. 2418970</i>			'5096	'220	8:10	'4886	'475	8:18	'4979.	'841	7:61
d.	P	m	'5118	'224	8:14	'4905	'479	8:22	'5058	'855	7:70
'2808	316'278	8:08	'5140	'228	8:15	'4924	'482	8:13	'5080	'859	7:65
'2827	'282	8:06	'5162	'232	8:12	'4942.	'485	8:16	'5102	'862	7:68
'2845	'285	8:08	'5184	'235	8:13	'4974	'491	8:27	'5124	'866	7:65
'2863	'288	8:06	'5206	'239	8:11	'4993	'494	8:22	'5155	'872	7:72
'2882	'291	8:20	'5229.	'243	8:10	'5011	'497	8:16	'5177	'876	7:60
'2900.	'295	8:23	'5303	'256	8:13	'5030	'501	8:16	'5199	'880	7:69
<i>J. D. 2418972</i>			'5325	'260	8:16	'5049	'504	8:18	'5221	'884	7:75
d.	P	m	'5347	'264	8:18	'5068	'507	8:15	'5243.	'887	7:65
'2717	319'791	7:59	'5370	'268	8:18	'5086	'510	8:08	<i>J. D. 2419300</i>		
'2736	'794	7:58	'5392	'272	8:16	'5104	'514	8:18	d.	P	m
'2754	'797	7:60	'5414	'276	8:18	'5123.	'517	8:10	'3781	898'620	7:38
'2772	'800	7:59	'5436	'280	8:21	'5206	'532	7:97	'3803	'624	7:32
'2790	'804	7:59	'5459.	'284	8:16	'5225	'535	7:87	'3825	'628	7:35
'2808	'807	7:66	'5490	'289	8:09	'5243	'538	7:85	'3847	'632	7:27
'2826	'810	7:60	'5513	'293	8:14	'5262	'542	7:86	'3868	'636	7:26
'2844.	'813	7:67	'5535	'297	8:11	'5281	'548	7:82	'3890	'639	7:33
'2865	'817	7:72	'5557	'301	8:23	'5315	'551	7:75	'3914	'643	7:29
'2888	'821	7:69	'5601	'309	8:25	'5333	'554	7:70	'3936.	'647	7:32
'2906	'824	7:73	'5623	'313	8:23	'5352.	'557	7:66	'3981	'655	7:25
'2924	'827	7:71	'5646.	'317	8:14	'5382	'563	7:66	'4003	'659	7:22
'2942	'830	7:68	<i>J. D. 2419275</i>			'5401	'566	7:68	'4027	'663	7:25
'2960	'834	7:68	d.	P	m	'5420	'569	7:54	'4048	'667	7:22
'2978	'837	7:64	'4716	854'681	7:13	'5438	'573	7:53	'4071	'671	7:27
'2996.	'840	7:68	'4738	'685	7:11	'5457	'576	7:50	'4091	'675	7:24
<i>J. D. 2418981</i>			'4760	'689	7:19	'5477	'580	7:49	'4114	'679	7:30
d.	P	m	'4783	'693	7:22	'5496	'583	7:43	'4135.	'682	7:16
'2687	335'663	7:11	'4808	'697	7:18	'5514.	'586	7:41	'4270	'706	7:18
'2706	'666	7:08	'4830	'701	7:21	<i>J. D. 2419280</i>			'4292	'710	7:25
'2724	'669	7:13	'4852	'705	7:21	d.	P	m	'4314	'714	7:19
'2743	'673	7:14	'4876.	'709	7:19	'4490	863'462	8:16	'4335	'718	7:18
'2762	'676	7:09	'4907	'715	7:23	'4505	'465	8:16	'4357	'722	7:25
'2780	'679	7:16	'4929	'719	7:24	'4527	'468	8:20	'4378	'725	7:23
'2798.	'682	7:13	'4951	'722	7:28	'4549	'472	8:16	'4400	'729	7:27
<i>J. D. 2418993</i>			'4973	'726	7:33	'4571	'476	8:12	'4422.	'733	7:23
d.	P	m	'4996	'730	7:35	'4593	'480	8:15	'4451	'738	7:34
'2185	356'744	7:47	'5018	'734	7:29	'4614	'484	8:12	'4473	'742	7:37
'2203.	'747	7:51	'5040	'738	7:36	'4636.	'488	8:10	'4494	'746	7:42
'2230	'752	7:42	'5062.	'742	7:40	'4665	'493	8:08	'4516	'750	7:38
'2248.	'755	7:39	'5178	'762	7:52	'4687	'497	8:09	'4538	'754	7:36
'2339	'771	7:42	'5200	'766	7:52	'4708	'500	8:04	'4560	'757	7:41
'2357	'774	7:33	'5221	'770	7:52	'4730	'504	8:11	'4582	'761	7:39
'2381	'778	7:48	'5240	'774	7:51	'4752	'508	8:04	'4604	'765	7:40
'2399.	'782	7:48	'5269	'779	7:44	'4774	'512	8:04	'4625.	'769	7:40
'2481	'796	7:47	'5291	'782	7:50	'4795	'516	8:03	<i>J. D. 2419326</i>		
'2499.	'799	7:54	'5313	'786	7:50	'4817.	'520	8:00	d.	P	m
'2523	'804	7:57	'5337	'791	7:53	'4843	'524	7:99	'3606	944'457	8:20
'2541.	'807	7:59	'5359.	'794	7:55	'4862	'528	7:92	'3628	'461	8:21
<i>J. D. 2419273</i>			<i>J. D. 2419276</i>			'4880	'531	7:96	'3650	'465	8:24
d.	P	m	d.	P	m	'4904	'535	7:90	'3672	'469	8:22
'4642	851'140	8:13	'4412	856'392	8:09	'4929	'539	7:91	'3694	'472	8:21
'4658	'142	8:17	'4430	'395	8:05	'4942.	'542	7:85	'3715	'476	8:14
<i>J. D. 2419283</i>			'4449	'398	8:07	<i>J. D. 2419283</i>			'3737	'480	8:16
d.	P	m	'4470	'402	8:14	d.	P	m	'3759.	'484	8:16
'4611	868'776	7:45	'4488	'405	8:12	'4611	868'776	7:45	'3794	'490	8:15
'4629	'779	7:61	'4507	'408	8:14	'4629	'779	7:61	'3816	'494	8:15

d	P	m	d	P	m	d	P	m	d	P	m
3838	498	8:17	4665	502	8:18	3427	576	7:37	3103	397	8:16
3859	502	8:14	4687	506	8:16	3449	580	7:32	3132	402	8:15
3874	504	8:08	4709	510	8:12	3471	584	7:20	3161	407	8:09
3896	508	8:13	4737	515	8:12	3493	588	7:19	3190	412	8:14
4044	534	7:95	<i>J. D. 2420006</i>			3515	592	7:13	3219	417	8:12
4066	538	7:94	d	P	m	3549	598	7:00	3249	422	8:10
4090	542	7:85	3861	2144 125	8:13	3571	602	7:04	3285	429	8:12
4112	546	7:88	3883	129	8:27	3593	606	6:99	3313	434	8:18
4134	550	7:75	3905	132	8:14	3615	610	6:98	3342	439	8:22
4149	553	7:73	3927	136	8:10	3637	614	6:97	3371	444	8:24
4170	556	7:73	3950	140	8:16	3658	617	6:96	3400	449	8:18
4192	560	7:67	3972	144	8:11	3719	628	6:83	3474	462	8:28
<i>J. D. 2420002</i>			4140	174	8:19	3741	632	6:88	3503	467	8:26
d	P	m	4162	178	8:21	3764	636	6:91	3532	472	8:18
4453	2137 172	8:11	4184	182	8:18	3786	640	6:90	3561	477	8:26
4475	176	8:18	4206	186	8:12	3808	644	6:93	3590	482	8:28
4499	181	8:14	4228	190	8:18	3831	648	6:95	3619	488	8:33
4521	185	8:02	4250	193	8:16	3864	654	6:94	3654	494	8:20
4543	188	8:21	4339	209	8:06	3896	659	7:01	3683	499	8:19
4565	192	8:18	4362	213	8:12	3918	663	6:95	3712	504	8:17
4596	198	8:22	4383	217	8:10	3940	667	6:97	3741	509	8:09
4618	202	8:15	4405	221	8:23	3963	671	6:99	3770	514	8:09
4640	206	8:11	4427	225	8:25	3985	675	7:07	3798	519	8:05
4662	210	8:06	4449	228	8:20	4076	691	7:02	3884	534	7:88
4684	213	8:19	4479	234	8:15	4098	695	7:10	3913	540	7:85
4706	217	8:13	4501	238	8:16	4120	699	7:15	3942	545	7:68
<i>J. D. 2420005</i>			4523	241	8:10	4142	703	7:12	3962	548	7:66
d	P	m	4545	245	8:23	4163	706	7:09	<i>J. D. 2420018</i>		
3482	2142 294	8:18	4567	249	8:13	4185	710	7:12	d	P	m
3503	297	8:12	4595	254	8:15	4220	716	7:22	3634	2165 255	8:02
3525	301	8:21	<i>J. D. 2420007</i>			4241	720	7:15	3672	261	8:07
3549	306	8:17	d	P	m	4263	724	7:24	3698	266	8:09
3570	309	8:16	3772	2145 873	7:60	4285	728	7:30	3728	271	8:10
3593	313	8:16	3794	877	7:70	4307	732	7:32	3756	276	8:06
3645	322	8:27	3816	881	7:68	4328	736	7:24	3785	281	8:07
3667	326	8:24	3838	885	7:67	4402	748	7:28	3846	292	8:03
3689	330	8:16	3860	889	7:66	4429	753	7:39	3874	297	8:02
3711	334	8:25	3882	893	7:70	4451	757	7:34	3907	303	8:00
3734	338	8:23	3915	898	7:71	4472	761	7:27	3939	308	8:03
3756	342	8:19	3936	902	7:80	4494	765	7:43	3967	313	8:00
3815	352	8:09	3958	906	7:72	4516	768	7:32	3996	318	7:98
3837	356	8:15	3980	910	7:72	4552	775	7:39	4045	327	8:01
3860	360	8:06	4003	914	7:78	4574	779	7:38	4074	332	8:08
3882	364	8:09	4025	918	7:82	4596	783	7:46	4102	337	8:08
3904	368	8:15	4084	928	7:79	4618	786	7:45	4131	342	8:04
3926	372	8:13	4109	933	7:89	4639	790	7:46	4160	347	7:99
3968	380	8:20	4136	937	7:89	4661	794	7:50	4189	352	8:04
3990	383	8:22	4157	941	7:88	<i>J. D. 2420012</i>			<i>J. D. 2420021</i>		
4013	387	8:22	4179	945	7:91	d	P	m	d	P	m
4035	391	8:16	4200	949	7:88	3112	2154 577	7:21	3098	2170 452	8:18
4057	395	8:15	4230	954	7:81	3142	583	7:18	3128	458	8:14
4072	398	8:18	4251	958	7:86	3171	588	7:12	3156	463	8:20
4151	412	8:19	4273	962	7:92	3200	593	7:11	3186	468	8:21
4173	416	8:05	4303	967	7:85	3229	598	7:04	3214	473	8:14
4195	420	8:10	4325	971	7:90	3258	603	7:02	3244	478	8:16
4217	423	8:11	4347	975	7:88	3295	610	6:96	3281	485	8:17
4239	427	8:14	4408	986	7:98	3325	615	7:04	3314	490	8:16
4262	431	8:22	4430	989	7:92	3354	620	7:06	3343	496	8:17
4289	436	8:14	4452	993	7:90	3383	625	7:05	3372	501	8:15
4318	441	8:09	4474	997	7:80	3412	630	6:96	3401	506	8:05
4340	445	8:16	4495	2146 001	7:96	3441	636	7:05	3430	511	8:03
4362	449	8:12	4517	005	7:99	3537	652	7:07	3466	517	7:98
4384	453	8:11	4551	011	7:82	3559	656	7:04	3495	522	7:98
4406	457	8:12	4573	014	7:88	3587	661	7:08	3524	528	7:95
4475	469	8:08	4595	018	7:92	3616	666	7:10	3553	533	7:93
4497	473	8:10	4617	022	7:92	3645	672	7:11	3582	538	7:84
4519	477	8:11	4638	026	7:91	3674	677	7:16	3610	543	7:82
4541	481	8:08	4660	030	7:96	3704	682	7:21	3646	549	7:68
4563	484	8:23	<i>J. D. 2420008</i>			3739	688	7:17	3678	555	7:59
4585	488	8:14	d	P	m	<i>J. D. 2420017</i>			3771	571	7:45
4620	494	8:24	3405	2147 572	7:41	d	P	m	3824	580	7:38
4643	498	8:07				3081	2163 393	8:11	3859	587	7:34

d	P	m	d	P	m	d	P	m	d	P	m
'3888	'592	7'20	'3628	'480	8'19	'4141.	'627	7'29	'3216	'520	8'13
'3916.	'597	7'27	'3658	'485	8'33	'4197	'637	7'17	'3245	'526	8'13
'3952	'603	7'24	'3686	'490	8'17	'4219	'641	7'20	'3274	'531	8'09
'3977	'607	7'25	'3716	'495	8'22	'4240	'644	7'17	'3303	'536	8'08
'3998	'611	7'24	'3745	'500	8'20	'4262	'648	7'15	'3332.	'541	8'06
'4020	'615	7'23	'3774.	'506	8'28	'4284	'652	7'12	'3384	'550	8'02
'4042	'619	7'23	'3809	'512	8'15	'4306.	'656	7'09	'3413	'555	8'02
'4067.	'623	7'23	'3839.	'517	8'20	'4357	'665	7'11	'3443	'560	7'92
'4101	'629	7'24	<i>J. D. 2420037</i>			'4379	'669	6'98	'3472	'566	7'88
'4126	'634	7'25	d	P	m	'4401	'673	7'10	'3500	'571	7'84
'4152	'638	7'21	'3226	2198'701	7'10	'4423	'677	6'99	'3529.	'576	7'78
'4173	'642	7'20	'3248	'705	7'09	'4444	'680	7'03	'3582	'585	7'66
'4195	'646	7'18	'3273	'710	7'15	'4466.	'684	7'05	'3611	'590	7'56
'4217.	'650	7'16	'3292	'713	7'13	'4516	'693	7'11	'3640	'595	7'48
'4246	'655	7'26	'3314	'717	7'16	'4546	'698	7'13	'3669	'600	7'39
'4267	'659	7'26	'3336.	'721	7'15	'4574	'703	7'13	'3703	'606	7'35
'4289	'662	7'27	'3365	'726	7'16	'4603	'708	7'10	'3732.	'611	7'29
'4311	'666	7'27	'3387	'730	7'17	'4632	'714	7'15	'3781	'620	7'17
'4333	'670	7'34	'3408	'734	7'19	'4661.	'719	7'15	'3810	'625	7'16
'4354.	'674	7'31	'3430	'737	7'16	<i>J. D. 2420040</i>			'3839	'630	7'13
'4378	'678	7'32	'3452	'741	7'20	d	P	m	'3868	'635	7'09
'4406	'683	7'30	'3474.	'745	7'14	'3239	2203'996	7'86	'3896	'640	7'09
'4427	'687	7'32	'3514	'752	7'24	'3267	2204'001	7'86	'3925.	'646	7'10
'4449	'691	7'31	'3542	'757	7'36	'3303	'007	7'91	'3985	'656	7'05
'4471	'695	7'33	'3571	'762	7'34	'3325	'011	7'85	'4014	'661	7'04
'4493.	'698	7'31	'3600	'767	7'26	'3354	'016	7'87	'4048	'667	7'00
<i>J. D. 2420023</i>			'3629	'772	7'32	'3383.	'022	7'93	'4078	'672	7'02
d	P	m	'3657.	'778	7'31	'3421	'028	7'82	'4107	'678	6'99
'3499	2174'051	7'96	<i>J. D. 2420038</i>			'3450	'033	7'90	'4136.	'683	7'05
'3524	'056	7'96	d	P	m	'3479	'038	7'96	<i>J. D. 2420062</i>		
'3550	'060	7'96	'3235	2200'467	8'39	'3508	'044	7'93	d	P	m
'3571	'064	8'03	'3264	'472	8'36	'3537	'049	7'94	'2826	2242'735	7'35
'3593	'068	8'01	'3292	'477	8'38	'3566.	'054	7'98	'2855	'740	7'43
'3615.	'072	7'97	'3321	'482	8'41	'3634	'066	7'96	'2884	'745	7'37
'3644	'077	8'14	'3350	'487	8'42	'3670	'072	8'04	'2913	'750	7'37
'3667	'081	8'06	'3379.	'492	8'43	'3706	'078	8'30	'2942	'755	7'49
'3688	'085	8'01	'3415	'499	8'12	'3748	'086	8'18	'2971.	'760	7'43
'3710	'089	8'01	'3444	'504	8'16	'3784	'092	8'11	'3016	'768	7'50
'3732	'092	8'03	'3472	'509	8'14	'3820.	'099	8'02	'3046	'773	7'56
'3754.	'096	8'07	'3501	'514	8'14	'3881	'109	8'03	'3074	'778	7'55
'3808	'106	7'98	'3530	'519	8'13	'3917	'116	8'06	'3103	'784	7'60
'3830	'110	8'09	'3559.	'524	8'08	'3953	'122	8'06	'3132	'789	7'61
'3844	'112	8'06	'3599	'531	8'18	'3989	'128	8'08	'3161.	'794	7'57
'3873	'117	7'99	'3628	'536	8'17	'4025	'135	8'08	'3235	'807	7'48
'3894	'121	8'06	'3657	'542	8'17	'4061.	'141	8'10	'3252	'810	7'44
'3916.	'125	8'12	'3686	'547	8'15	<i>J. D. 2420042</i>			'3281	'815	7'50
'3945	'130	8'03	'3715	'552	8'11	d	P	m	'3310	'820	7'55
'3967	'134	7'97	'3744.	'557	8'07	'2798	2207'447	8'22	'3339	'825	7'54
'3989	'138	8'05	'3794	'566	7'93	'2828	'452	8'18	'3367.	'830	7'54
'4030	'145	8'06	'3823	'571	7'90	'2857	'457	8'16	'3425	'840	7'58
'4052	'149	8'07	'3852	'576	7'89	'2885	'462	8'15	'3456	'846	7'69
'4074.	'153	8'07	'3881	'581	7'77	'2915	'467	8'18	'3485	'851	7'57
<i>J. D. 2420034</i>			'3911	'586	7'79	'2944.	'472	8'19	'3517	'856	7'63
d	P	m	'3940.	'591	7'75	'2979	'479	8'18	'3546	'862	7'62
'3445	2193'448	8'18	'3976	'598	7'52	'3008	'484	8'16	'3574.	'867	7'61
'3476	'453	8'18	'4005	'603	7'53	'3038	'489	8'25	'3627	'876	7'72
'3505	'458	8'18	'4034	'608	7'46	'3067	'494	8'22	'3656	'881	7'67
'3535	'463	8'20	'4063	'613	7'41	'3096	'499	8'24	'3685	'886	7'71
'3564	'468	8'20	'4092	'618	7'33	'3125.	'504	8'22	'3714	'891	7'71
'3593.	'474	8'23	'4113	'622	7'34	'3185	'515	8'13	'3742	'896	7'70
									'3771.	'901	7'69

APPENDIX. (Additional Observations).

Previously to the observations discussed above I obtained 11 plates on 5 nights with 78 exposures altogether, which were taken in the other position of the instrument viz. with the telescope to the west of the pier. The dates of these plates are given in Table 7 and the individual observations in Table 8. The apparent

TABLE 7

Date	Nr. of first plate	Sid. time Potsdam middle		Number of		<i>J. D.</i>	Phase observed	
		first exp.	last exp.	plates	exp.		first exp.	last exp.
		h m	h m				P	P
1910 May 1	238	16 35	16 58	1	8	2418793	4'493	5'21
3	241	14 41	17 10	5	36	8795	7'872	0'53
5	246	14 37	15 37	2	16	8797	11'386	4'59
7	254	14 36	14 43	1	3	8799	14'903	9'12
9	250	14 54	15 52	2	15	8801	18'444	5'14

magnitudes of the comparison stars were again derived from 6 other plates taken with the grating in front of the objective. When *ef* (the mean of stars *e* and *f*) is as above taken to be 8^m.000, I find for *c* 7^m.283 and for *ghx* 8^m.310. According to the corrections for position on the plate as determined by Prof. MÜNCH, and also taking into account the differences in differential extinction of the stars in the two series, the apparent magnitude to be adopted for *ef* comes out to be practically the same for these additional observations as for those already discussed. The magnitudes given in Table 8 ought accordingly to be directly

comparable with those in Table 3. Nevertheless I have thought it safer to treat them separately in order not to increase the risk of inhomogeneity of the material. On the other hand I would not entirely suppress these 78 observations. Those from *J. D.* 2418801 or 18^P are shown by open circles on the first of the 9 diagrams. They show an early rise to maximum light, the same as the observations from 226^P or about 3 × 71^P.64 later.

Their total weight, calculated as described above, is 5670 *m*⁻².

TABLE 8.

<i>J. D.</i> 2418793			<i>J. D.</i> 2418795			<i>J. D.</i> 2418797			<i>J. D.</i> 2418799			<i>J. D.</i> 2418801		
d	P	m	d	P	m	d	P	m	d	P	m	d	P	m
5467	4'493	8'11	4935	9'27	7'82	5605	0'45	8'12	4475	14'903	7'80	4550	18'444	8'10
5491	4'97	8'13	4957	9'31	7'81	5628	0'50	8'11	4497	9'07	7'85	4585	4'50	8'05
5513	5'01	8'08	4979	9'35	7'83	5650	0'53	8'15	4526	9'12	7'87	4620	4'56	8'13
5536	5'05	8'08	5001	9'39	7'84	<i>J. D.</i> 2418797			4642	4'60	8'07	4642	4'60	8'07
5558	5'09	8'06	5022	9'43	7'86	d	P	m	4663	4'64	8'13	4663	4'64	8'13
5581	5'13	8'04	5098	9'56	7'92	4542	11'386	8'11	4684	4'68	8'15	4684	4'68	8'15
5603	5'17	8'04	5122	9'60	7'97	4566	3'90	8'11	4708	4'72	8'07	4708	4'72	8'07
5625	5'21	8'05	5145	9'64	7'94	4591	3'95	8'08	4792	4'87	8'05	4792	4'87	8'05
			5167	9'68	7'94	4614	3'99	8'11	4816	4'91	8'07	4816	4'91	8'07
			5191	9'72	8'00	4636	4'03	8'10	4838	4'95	8'06	4838	4'95	8'06
			5213	9'76	7'99	4658	4'07	8'09	4860	4'99	7'96	4860	4'99	7'96
			5235	9'80	7'94	4680	4'10	8'15	4882	5'03	7'92	4882	5'03	7'92
			5264	9'85	7'92	4702	4'14	8'13	4904	5'07	7'88	4904	5'07	7'88
			5343	9'99	7'87	4798	4'31	8'13	4926	5'10	7'82	4926	5'10	7'82
			5365	8'003	7'92	4820	4'35	8'08	4948	5'14	7'79	4948	5'14	7'79
			5387	0'07	7'94	4843	4'39	8'21						
			5409	0'11	8'01	4865	4'43	8'11						
			5431	0'15	8'00	4887	4'47	8'04						
			5454	0'19	8'02	4909	4'51	8'14						
			5476	0'23	8'06	4931	4'55	8'10						
			5498	0'26	8'05	4953	4'59	8'13						
			5582	0'41	8'08									