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COMMUNICATIONS FROM THE BOSSCHA OBSERVATORY AT LEMBANG.

The double stars in the Greenwich Astrographic Catalogue, third paper

by Dr. *E. A. Kreiken.*

The double stars contained in the different zones of the Greenwich Astrographic Catalogue have been collected by Dr. H. GROOT. Until now only a short account of his results has been published (*M. N. R. A. S.* Vol. LXXXVIII No. 1 pages 51—56). The Astronomer Royal and Dr. GROOT gave me permission to use the full results of his catalogue in advance of publication. I am very much indebted to Sir FRANK DYSON and Dr. GROOT for their extreme kindness.

As the magnitudes in the Greenwich Astrographic Catalogue are much more carefully determined than in most of the other Carte du Ciel zones, the results of this investigation were greatly improved by taking in account this additional material.

In the present paper I have discussed the double stars in the zones of declination 64°, 65°.... and 72°.

The method of reduction used here is the same as in the preceding articles (*B. A. N.* **151**), so only a few explanations will be required here.

Table 1 gives a detailed account of the distribution of the observed number of double stars over m and Δm in the different zones of galactic latitude $\beta < 20°$, from 20° to 40°, from 40° to 50° and $> 50°$.

As before the magnitude of a double star was taken equal to that of the brighter component. Δm indicates the magnitude difference between the two components of the pair.

As the values in table 1 indicate the distribution of the observed numbers of double stars, i. e. physical + optical numbers, the distribution of the physical pairs will be found by substracting from the values of table 1 the numbers of optical pairs.

In the first place we determine the numbers of optical pairs between definite magnitude limits m_1 and m_2.

Therefore we use the formula (1)

$$D_0(m) = \frac{d^2}{r^2}\left[\frac{A(m)\,[A(m)-1]}{2} + N(m_1) - N(m_2)\right] \quad (1)$$

$A(m)$ indicates the observed number of stars in the magnitude interval m_1 to m_2, $N(m_1)$ is the total number of stars up to the limitting magnitude m_2 of the catalogue and $N(m_2)$ the total number of stars up to the upper limit m_2 of the magnitude interval.

d is the distance in seconds of arc between the two components of a pair and r the radius of a circular area of the sky with a surface equal to the surface of the sky considered here.

Before applying formula (1) the plates in the different zones of galactic latitude were devided into four separate groups. The first group contains the plates which are complete down to a magnitude fainter than 13$^{\text{th}}$ magnitude (Greenwich scale), the second one contains the plates complete down to at least $m = 12.5$, the third to $m = 12.0$, while the last group contains those plates which have a limiting magnitude $< 12.0^{\text{th}}$ magnitude. It was possible to devide the plates in these separate groups by using the tables on page 177 and following of the Greenwich Astrographic Catalogue Vol. IV (proper motions and photographic magnitudes).

Formula (1) was applied to the numbers derived from each separate group. From the separate sets of values a weighted mean was derived. The weight given to the values of each group was always taken equal to the number of plates contained in the group. In table 2 I have compared these mean values (optical numbers) with the observed numbers of double stars. We find the same results as in the preceding papers. The percentages of the double stars which are purely optical increase with increasing magnitude and with the larger values of d. In the group $d = 15''$—$30''$ nearly all pairs are optical ones.

The irregularities in the values of the limitting magnitude of the separate plates have a considerable influence on the computed numbers of optical pairs. This appears from table 3 were I have given the ratios of the numbers of optical pairs derived from the separate groups of plates and the numbers of optical pairs derived from the mean star numbers on all plates together.

TABLE 1. Distribution of the observed numbers in the Greenwich Astrographic Catalogue between the Declinatione 64° and 72°; m and Δm on the scale of the Greenwich Catalogue.

$\beta = 0° - 20°$

$d < 5''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
< 7.0	0	0	0	0	0	0	0	0	0
7.0 – 8.0	0	0	0	0	0	0	0	0	0
8.0 – 9.0	0	0	0	0	0	0	0	0	1
9.0 – 10.0	0	0	2	1	0	1	1	1	3
10.0 – 11.0	4	4	3	5	1	4	1	1	1
11.0 – 12.0	12	6	8	2	2	1	0	1	0
12.0 – 13.0	16	6	3	1	0	1			
13.0 >	3								

$d = 5'' - 10''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
< 7.0	0	0	0	0	0	0	0	0	1
7.0 – 8.0	0	0	0	0	0	0	0	0	0
8.0 – 9.0	0	0	0	0	0	1	0	0	11
9.0 – 10.0	3	0	2	0	4	2	0	1	7
10.0 – 11.0	4	5	1	5	2	2	2	1	2
11.0 – 12.0	14	7	6	4	4	4	1	1	
12.0 – 13.0	11	3	2	2	1				
13.0 >	6	1							

$d = 10'' - 15''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
< 7.0	0	0	0	0	0	0	0	0	1
7.0 – 8.0	0	0	0	0	0	1	0	0	3
8.0 – 9.0	0	0	1	0	0	0	1	2	1
9.0 – 10.0	1	1	1	0	1	1	0	0	5
10.0 – 11.0	7	2	0	1	1	3	4	3	6
11.0 – 12.0	3	4	6	2	5	2	0	1	2
12.0 – 13.0	10	5	3	2					
13.0 >	4	1	1						

$d = 15'' - 30''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
< 7.0	1	0	0	0	0	0	0	0	2
7.0 – 8.0	0	0	0	0	0	1	0	0	2
8.0 – 9.0	0	0	4	0	1	2	2	2	8
9.0 – 10.0	1	0	1	3	3	2	4	1	11
10.0 – 11.0	4	3	7	11	5	4	11	4	5
11.0 – 12.0	21	13	17	7	13	11	3	1	
12.0 – 13.0	31	21	12	3	4	2	—		
13.0 >	21	6	1						

$\beta = 20° - 40°$

$d < 5''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
< 7.0	0	0	0	0	0	0	0	0	0
7.0 – 8.0	0	0	0	0	0	0	0	0	0
8.0 – 9.0	0	1	0	0	0	0	0	0	1
9.0 – 10.0	2	0	1	1	0	0	1	0	2
10.0 – 11.0	1	5	0	2	1	0	0	1	1
11.0 – 12.0	6	1	6	2	2	2			
12.0 – 13.0	8	3	2	1					
13.0 >	1	0	1						

$d = 5'' - 10''$

m \ Δm	0.0	0.3	0.6	0.9	1.7	1.5	1.8	2.1	2.4 >
< 7.0	0	0	0	0	0	1	0	0	0
7.0 – 8.0	0	0	0	0	0	0	0	0	0
8.0 – 9.0	0	1	0	0	0	0	0	0	1
9.0 – 10.0	0	1	0	1	1	0	2	1	5
10.0 – 11.0	0	4	1	2	2	1	2	1	3
11.0 – 12.0	2	6	3	1	0	0	1	1	
12.0 – 13.0	10	7	2	1	1				
13.0 >	1								

TABLE I (*Continued.*)

$d = 10'' - 15''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
– 7.0	0	0	0	0	0	0	0	0	1
7.0 – 8.0	1	0	0	0	0	0	0	0	1
8.0 – 9.0	0	0	0	0	1	0	0	0	1
9.0 – 10.0	0	1	0	0	1	0	0	0	2
10.0 – 11.0	3	0	0	2	2	0	0	0	1
11.0 – 12.0	3	3	2	3	4	1	1	0	0
12.0 – 13.0	4	4	1	2					
13.0 >	3	2							

$d = 15'' - 30''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
– 7.0	0	0	0	0	0	0	0	0	4
7.0 – 8.0	1	0	0	0	0	0	0	0	3
8.0 – 9.0	0	0	0	0	0	0	0	0	8
9.0 – 10.0	1	0	1	0	1	2	0	4	10
10.0 – 11.0	2	4	2	4	2	5	6	9	6
11.0 – 12.0	8	5	5	8	7	2	0	0	0
12.0 – 13.0	23	18	9	1	1				
13.0 >	12	2							

β 40° — 50°

$d < 5''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
– 7.0	0	0	0	0	0	0	0	0	0
7.0 – 8.0	0	0	0	0	0	0	0	0	0
8.0 – 9.0	0	0	0	0	0	0	0	0	0
9.0 – 10.0	1	0	0	1	0	0	1	0	2
10.0 – 11.0	0	0	0	1	0	0	2	0	2
11.0 – 12.0	4	2	0	2	0	0	1		
12.0 – 13.0	5	2	2						
13.0 >	0	1							

$d = 5'' - 10''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
– 7.0	0	0	0	0	0	0	0	0	0
7.0 – 8.0	0	0	0	0	0	0	0	0	0
8.0 – 9.0	0	0	0	0	0	0	0	0	2
9.0 – 10.0	0	1	0	0	0	0	0	1	0
10.0 – 11.0	3	0	0	2	1	1	1	1	0
11.0 – 12.0	1	1	0	3	1	1	1	1	
12.0 – 13.0	7	3							
13.0 >	3								

$d = 10'' - 15''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
– 7.0	0	0	0	0	0	0	0	0	0
7.0 – 8.0	0	0	0	0	0	0	0	0	0
8.0 – 9.0	1	0	0	0	0	0	0	0	2
9.0 – 10.0	0	1	0	0	0	0	0	0	0
10.0 – 11.0	0	0	0	1	0	1	1	3	0
11.0 – 12.0	1	0	1	2	0	2	1	0	0
12.0 – 13.0	2	2	0	1	0	1			
13.0 >	3								

$d = 15'' - 30''$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
– 7.0	0	1	0	0	0	0	0	0	1
7.0 – 8.0	0	0	0	0	0	1	0	0	0
8.0 – 9.0	0	0	0	0	0	0	1	0	2
9.0 – 10.0	1	0	0	0	2	0	0	1	5
10.0 – 11.0	0	1	1	2	1	2	3	2	5
11.0 – 12.0	2	3	4	3	3	1			
12.0 – 13.0	12	7	2	3	1	3			
13.0 >	6	2							

TABLE 1 (*Continued.*)

60° > β > 50°

m \ Δm	*d* < 5": 0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	*d* = 5"—10": 0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	3
9.0	0	1	0	0	0	0	0	0	1	0	1	0	0	1	0	0	1	1
10.0	0	2	0	0	1	1	0	0	0	2	1	0	0	0	1	1	0	1
11.0	2	0	0	2	0	1	1	0	1	2	1	1	0	1	2			
12.0	2	1	2	2						1	2	2						
13.0 >	3									1								

m \ Δm	*d* = 10"—15": 0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	*d* = 15"—30": 0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
7.0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
8.0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	1	1	1
9.0	0	1	0	0	1	0	0	1	1	0	0	0	0	0	0	3	0	2
10.0	2	1	0	0	0	1	1	0	1	0	3	2	1	0	3	1	2	0
11.0	2	1	1	0	1	2	0			3	0	2	5	1	2	0	1	1
12.0	1	2	2							9	2	4	3	2				
13.0 >	1									3	0	1						

TABLE 2. Comparison of the observed number of double stars with the numbers of optical pairs.

β = 0 — 20 surface 480 □ degrees.

m Gr.	*d* < 5"				*d* = 5"—10"				*d* = 10"—15"				*d* = 15"—30"			
	$D(m)$ Obs.	$D_0(m)$ Opt.	Diff. = Phys.	% Phys.	$D(m)$ Obs.	$D_0(m)$ Opt.	Diff. = Phys.	% Phys.	$D(m)$ Obs.	$D_0(m)$ Opt.	Diff. = Phys.	% Phys.	$D(m)$ Obs.	$D_0(m)$ Opt.	Diff. = Phys.	% Phys.
	0	0	0	—	1	0	1	—	1	0	1	100	3	2	1	33
7.0	0	0	0	—	0	0	0	—	4	1	3	75	3	4	(— 1)	0
8.0	1	0	1	100	12	1	11	92	5	2	3	60	19	12	+ 7	37
9.0	9	1	8	89	19	4	15	79	10	6	4	40	26	34	— 8	0
10.0	24	3	21	88	24	7	17	71	27	12	15	56	54	65	— 11	0
11.0	32	4	28	87	41	11	30	73	25	18	7	28	86	99	— 13	0
12.0	27	3	24	89	19	8	11	58	20	14	6	30	75	76	— 1	0
13.0 >	3	0	3	100	7	1	6	86	6	1	5	83	28	8	+ 20	71

TABLE 2 (*Continued*).

$\beta = 20^\circ - 40$ surface 288 □ degrees.

m Gr.	$d < 5''$				$d = 5'' - 10''$				$d = 10'' - 15''$				$d = 15'' - 30''$			
	$D(m)$ Obs.	$D_0(m)$ Opt.	Diff. = Phys.	°/° Phys.	$D(m)$ Obs.	$D_0(m)$ Opt.	Diff. = Phys.	°/° Phys.	$D(m)$ Obs.	$D_0(m)$ Opt.	Diff. = Phys.	°/° Phys.	$D(m)$ Obs.	$D_0(m)$ Opt.	Diff. = Phys.	°/° Phys.
7.0	0	0	0	—	1	0	1	100	1	0	1	100	4	1	+ 3	75
8.0	0	0	0	—	0	0	0	—	2	0	2	100	4	2	+ 2	50
9.0	2	0	2	100	2	0	2	100	2	1	1	50	8	5	+ 3	38
10.0	7	0	7	100	11	1	10	91	4	2	2	50	19	12	+ 7	37
11.0	11	1	10	91	16	4	12	75	8	6	2	25	40	34	+ 6	15
12.0	19	2	17	90	14	5	9	64	17	9	8	47	35	51	— 16	0
13.0	14	1	13	93	21	4	17	81	11	6	5	45	52	34	+ 18	35
>	2	0	2	100	1	0	1	100	5	1	4	80	14	3	+ 11	79

$\beta = 40^\circ - 50$ surface 192 □ degrees.

m Gr.	$d < 5''$				$d = 5'' - 10''$				$d = 10'' - 15''$				$d = 15'' - 30''$			
	$D(m)$ Obs.	$D_0(m)$ Opt.	Diff. = Phys.	°/° Phys.	$D(m)$ Obs.	$D_0(m)$ Opt.	Diff. = Phys.	°/° Phys.	$D(m)$ Obs.	$D\ (m)$ Opt.	Diff. = Phys.	°/° Phys.	$D(m)$ Obs.	$D_0(m)$ Opt.	Diff. = Phys.	°/° Phys.
7.0	0	0	0	—	0	0	0	—	0	0	0	—	2	0	2	100
8.0	0	0	0	—	0	0	0	—	0	0	0	—	1	1	0	0
9.0	0	0	0	—	2	0	2	100	3	0	3	100	3	2	1	33
10.0	5	0	5	100	2	0	2	100	1	1	0	0	9	4	5	56
11.0	5	0	5	100	9	1	8	89	6	2	4	67	17	10	7	41
12.0	9	1	8	89	9	2	7	78	7	3	4	57	16	18	(— 2)	0
13.0	9	1	8	89	10	2	8	80	6	3	3	50	28	17	11	39
>	1	0	1	100	3	0	3	100	3	1	2	67	8	3	5	63

$60^\circ > \beta > 50^\circ$ surface 192 □ degrees.

m Gr.	$d < 5''$				$d = 5'' - 10''$				$d = 10'' - 15''$				$d = 15'' - 30''$			
	$D(m)$ Obs.	$D_0(m)$ Opt.	Diff. = Phys.	°/° Phys.	$D(m)$ Obs.	$D_0(m)$ Opt.	Diff. = Phys.	°/° Phys.	$D(m)$ Obs.	$D_0(m)$ Opt.	Diff. = Phys.	°/° Phys.	$D(m)$ Obs.	$D_0(m)$ Opt.	Diff. = Phys.	°/° Phys.
7.0	0	0	0	—	0	0	0	—	0	0	0	—	1	0	1	100
8.0	0	0	0	—	0	0	0	—	1	0	1	100	0	1	(— 1)	0
9.0	2	0	2	100	3	0	3	100	2	0	2	100	3	2	1	33
10.0	2	0	2	100	4	1	3	75	4	1	3	75	5	5	0	0
11.0	4	0	4	100	6	1	5	83	6	2	4	67	12	12	0	0
12.0	7	1	6	86	7	2	5	71	7	3	4	56	15	15	0	0
13.0	7	1	6	86	5	2	3	60	5	3	2	40	20	17	3	15
>	3	0	3	100	1	0	1	100	1	0	1	100	4	1	3	75

TABLE 3. Ratio: $\dfrac{\text{number of optical pairs computed from the separate groups of plates}}{\text{number of optical pairs computed from the mean of all plates together}}$

m Gr \ β	0—20	20—40	40—50	> 50	m International 0—20	20—40	40—50	> 50
	1.26	1.22	1.25	1.19				
7.0					7.15	7.37	7.45	7.45
	1.24	1.17	1.17	1.12				
8.0					8.17	8.43	8.62	8.62
	1.23	1.18	1.19	1.15				
9.0					9.24	9.42	9.75	9.75
	1.28	1.24	1.25	1.09				
10.0					10.26	10.50	10.56	10.56
	1.35	1.22	1.23	1.15				
11.0					11.27	11.62	11.53	11.53
	1.49	1.38	1.35	1.34				
12.0					12.41	12.60	12.51	12.51
	3.66	2.69	2.24	1.76				
13.0					13.20	13.38	13.33	13.33
>	—	8.43	6.27	20.—				
Mean	1.70	1.51	1.55	1.36				

TABLE 4. Limiting magnitude of the different groups of plates and numbers of plates in the different groups.

β	0—20°	20—40°	40—50°	> 50°
magn. limit.	13.27 . 12.59 . 12.24 . 11.90 .	13.66 . 13.35 . 13.21 . 12.35 .	13.81 . 13.53 . 12.76 . 12.36 .	14.12 . 13.26 . 12.94 .
plate numbers	31 42 26 6	21 27 14 3	18 11 11 1	15 15 8

The numbers of plates involved in each group and the statistical limiting magnitude (on the international photographic scale) have been entered in table 4.

It should be directly remarked that the values of Dr. GROOT (*l. c.* page 53) are not immediately comparable with mine, as the values derived by Dr. GROOT are based on the mean numbers of stars in the whole Astrographic Catalogue.

In table 3 I have also given the international photographic magnitude corresponding to the magnitude limits adopted here.

They were obtained by comparing the values of the table on page A13 (Greenw. Astr. Cat. Vol. IV) with the values in the tables published by SEARES, VAN RHIJN, JOYNER and RICHMOND. (*The Astr. phys. Journ.* Vol. LXII. page 320 and following). For the latitudes $\beta = 40$—50 and $\beta > 50°$ we have used the same magnitude scale, as β is never larger than 56°.

The distribution of the numbers of optical pairs over different values of Δm is given in table 5. Up to the magnitude $m_g = 11.0$ the fractions contained in this table were computed from the formula (2)

$$D_0(m, \Delta m) = \frac{\displaystyle\int_{x=\left(+\frac{\beta}{2\gamma}+\overline{m}_g\right)\sqrt{\gamma}}^{x=\left(\Delta m+\frac{\beta-2\gamma\overline{m}_g}{2\gamma}\right)\sqrt{\gamma}} e^{-x^2}\,dx}{\displaystyle\int_{x=\left(+\frac{\beta}{2\gamma}+\overline{m}_g\right)\sqrt{\gamma}}^{x=\left(\Delta m_l+\frac{\beta-2\gamma\overline{m}_g}{2\gamma}\right)\sqrt{\gamma}} e^{-x^2}\,dx} \times D_0(m) \,.. \quad (2)$$

The upper and lower limit of the integrals in formula (2) depend on the numerical values of β and γ, the coefficients of KAPTEYN's apparent luminosity curve. For the different galactic zones considered here, these values have been determined in the preceding article. The same values were used here.

Formula (2) was first applied to the separate groups of plates mentioned before. This is necessary as the values of the fractions (2) will be largely influenced by the limitting magnitude m_l. The values in Table 5 represent the weighted mean, obtained from the different sets of values derived from each group of plates.

If it is argued, that all fainter double stars with an angular separation $d = 15''-30''$ are optical ones, the observed distribution of table I also is the Δm distribution of the optical pairs. So the last three rows of values in table 5 were determined by simply dividing the observed numbers in a certain interval of m and Δm by the total number of double stars in the same magnitude interval.

We will find the distribution of the physical pairs over m and Δm by substracting from the observed numbers in our table 1, the numbers of optical pairs in each m, Δm interval.

These optical numbers are found from our second and fifth tables.

We multiply the fractions in table 5 by the total number of optical pairs $D_0(m)$ given in table 2.

The final results derived in this way have been entered in table 6.

TABLE 5. Values of 100 × fractions for computing the distribution of the optical pairs over Δm (m and Δm on the scale of the Greenwich Astrographic Catalogue).

$\beta = 0°$-$20°$ surface 480 □ degrees

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
< 7.0	0.2	0.1	0.1	0.3	0.4	0.4	0.3	0.9	97.3
7.0–8.0	0.6	0.3	0.3	0.9	1.2	1.2	0.9	2.9	91.7
8.0–9.0	0.9	1.2	1.2	1.5	3.5	3.8	4.4	5.0	78.7
9.0–10.0	3.1	3.1	4.0	5.5	8.9	8.0	15.9	12.4	39.1
10.0–11.0	4.7	10.3	14.7	19.9	15.3	17.2	7.8	6.0	4.1
11.0–12.0	24.5	15.1	19.8	8.2	15.1	12.8	3.5	1.0	—
12.0–13.0	42.5	28.8	16.4	4.1	5.5	2.7	—	—	—
13.0 >	75.0	21.4	3.6	—	—	—	—	—	—

$\beta = 20°$-$40°$ surface 288 □ degrees

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
< 7.0	0.1	0.1	0.2	0.2	0.3	0.5	0.7	0.8	97.1
7.0–8.0	0.3	0.4	0.7	0.8	1.0	1.5	2.1	2.6	90.6
8.0–9.0	0.6	0.7	1.0	1.3	1.9	2.4	3.3	4.2	84.6
9.0–10.0	1.6	2.0	2.8	3.7	4.8	6.5	8.5	10.1	60.0
10.0–11.0	5.0	7.0	9.0	11.6	15.4	17.6	22.8	2.2	9.4
11.0–12.0	22.9	14.3	14.3	22.9	20.0	5.6	—	—	—
12.0–13.0	44.4	34.6	17.3	1.9	1.8	—	—	—	—
13.0 >	85.7	14.3	—	—	—	—	—	—	—

$\beta = 40°$-$50°$ 192 □ degrees

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
< 7.0	0.0	0.1	0.1	0.2	0.2	0.3	0.4	0.6	98.1
7.0–8.0	0.2	0.4	0.4	0.6	0.7	1.0	1.3	1.9	93.5
8.0–9.0	0.8	1.2	1.4	2.0	2.0	3.2	4.2	5.6	79.6
9.0–10.0	2.1	2.6	3.6	4.7	6.0	7.9	10.2	13.0	49.9
10.0–11.0	5.5	7.1	9.3	12.0	15.3	6.8	12.0	9.8	22.2
11.0–12.0	16.9	14.6	19.1	11.1	10.3	14.9	13.1	—	—
12.0–13.0	42.9	25.0	7.2	10.8	3.6	10.5	—	—	—
13.0 >	75.0	25.0	—	—	—	—	—	—	—

$\beta > 50°$ 192 □ degrees

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
< 7.0	0.0	0.1	0.1	0.1	0.2	0.3	0.4	0.5	98.3
7.0–8.0	0.2	0.3	0.3	0.5	0.7	0.9	1.2	1.7	94.2
8.0–9.0	0.8	1.1	1.4	2.0	2.0	3.2	4.1	5.4	80.0
9.0–10.0	1.9	2.4	3.2	4.2	5.4	7.1	9.2	11.2	55.4
10.0–11.0	4.3	5.5	7.3	9.4	12.0	11.4	19.2	7.6	25.3
11.0–12.0	15.0	12.9	21.6	13.1	11.1	2.6	1.1	3.1	19.5
12.0–13.0	45.0	10.0	20.0	15.0	10.0				
13.0 >	75.0	25.0							

TABLE 6. Distribution of the physical pairs over m and Δm (m and Δm on the scale of the Greenwich Astrographic Catalogue)

$\beta = 0° - 20°$

	$d < 5''$									$d = 5'' - 10''$								
m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
< 7.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
7.0–8.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8.0–9.0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	10
9.0–10.0	0	0	2	1	0	1	1	1	2	3	0	2	0	4	2	(−1)	0	5
10.0–11.0	4	4	3	4	0	3	1	1	1	4	4	0	3	1	1	1	1	2
11.0–12.0	11	5	7	2	1	1	0	1	—	11	5	3	4	2	3	1	1	—
12.0–13.0	15	5	2	1	0	1	—	—	—	8	1	1	1	0	—	—	—	—
13.0 >	3	—	—	—	—	—	—	—	—	5	1	—	—	—	—	—	—	—

	$d = 10'' - 15''$								
m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
< 7.0	0	0	0	0	0	0	0	0	1
7.0–8.0	0	0	0	0	0	1	0	0	2
8.0–9.0	0	0	1	0	0	0	1	1	0
9.0–10.0	1	1	1	0	0	0	(−1)	(−1)	3
10.0–11.0	6	1	(−1)	0	0	1	3	2	3
11.0–12.0	(−1)	1	2	0	2	0	0	1	2
12.0–13.0	4	1	0	1	—	—	—	—	—
13.0 >	3	1	1						

$\beta = 20° - 40°$

	$d < 5''$									$d = 5'' - 10''$								
m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
< 7.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
7.0–8.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8.0–9.0	0	1	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1
9.0–10.0	2	0	1	1	0	0	1	0	2	0	1	0	1	1	0	2	1	4
10.0–11.0	1	5	0	1	1	0	0	1	1	0	4	1	1	1	0	2	1	3
11.0–12.0	5	1	5	2	2	2	—	—	—	1	5	2	0	(−1)	0	1	1	—
12.0–13.0	7	3	2	1						8	6	1	1	1	—	—	—	—
13.0 >	1	0	1							1								

TABLE 6 (*Continued*).

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
	$d = 10'' - 15''$								
	0	0	0	0	0	0	0	0	1
7.0									
	1	0	0	0	0	0	0	0	1
8.0									
	0	0	0	0	1	0	0	0	0
9.0									
	0	1	0	0	1	0	0	0	0
10.0									
	3	0	(—1)	1	1	(—1)	(—1)	0	0
11.0									
	1	2	1	1	2	0	1		
12.0									
	1	2	0	2					
13.0 >									
	2	2							

$\beta = 40° - 50$

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
	$d < 5''$									$d = 5'' - 10''$								
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7.0																		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8.0																		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
9.0																		
	1	0	0	1	0	0	1	0	2	0	1	0	0	0	0	0	1	0
10.0																		
	0	0	0	1	0	0	2	0	2	3	0	0	2	1	1	1	1	(— 1)
11.0																		
	3	2	0	2	0	0	1	—	—	0	1	0	2	1	1	1	1	
12.0																		
	4	2	2							6	2	0						
13.0																		
	0	1								3								

m \ Δm	0.0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4 >
	$d = 10'' - 15''$								
	0	0	0	0	0	0	0	0	0
7.0									
	0	0	0	0	0	0	0	0	0
8.0									
	1	0	0	0	0	0	0	0	2
9.0									
	0	1	0	0	0	0	0	0	0
10.0									
	0	0	0	0	0	1	1	2	0
11.0									
	0	0	0	2	0	1	1		
12.0									
	1	1	0	0	0	1			
13.0 >									
	2								