On the analysis of the light-variation of RV Tauri

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On the analysis of the light-variation of RV Tauri, by *Miss H. A. Kluyver*.

Introduction.

RV Tauri belongs to a group of variable stars with intermediate periods and complex light curves, the RV Tauri stars [1]). In the period of 78·534 days [2]) the brightness has two maxima and two minima. The light-variation shows a second period of about 1200 days [3]); moreover, Dr J. VAN DER BILT has found, on inspection of the minima minimorum, a third period, of roughly 8000 days [4]).

1. The present investigation is based on the following considerations, which have been set forth more amply by Dr J. WOLTJER JR [1]). According to the pulsation theory the brightness of a star which oscillates in three of its fundamental modes is a periodic function, with period 2π, of each of the three phase-arguments ω, ω' and ω''. For a star such as RV Tauri, where the occurrence of the long periods indicates the existence of approximate commensurabilities between the fundamental frequencies, it is appropriate to introduce, instead of the arguments ω' and ω'', arguments τ and ψ corresponding to these commensurabilities. Hence the brightness may be represented by a series of the form:

$$\sum_{0}^{+\infty} \sum_{-\infty}^{+\infty} \sum_{-\infty}^{+\infty} A_{ijk} \cos (i\omega + j\tau + k\psi) + \sum_{0}^{+\infty} \sum_{-\infty}^{+\infty} \sum_{-\infty}^{+\infty} B_{ijk} \sin (i\omega + j\tau + k\psi);$$

ω, τ and ψ correspond to the period of 78·534 days and to the second and third periods respectively. The various characteristics of the light curve, e.g. brightness at maximum and minimum, mean magnitude, phase of maximum and minimum, will be periodic functions of τ and ψ.

The threefold series may be derived as follows. Intervals of time are considered which are short as compared with the third period. For each of these intervals a double series, with arguments ω and τ only, is determined from the observations, the change of ψ being neglected; the coefficients of these series are in their turn functions of ψ, and after determination of these functions the threefold series is known. The computation of the double series, with arguments ω and τ, is described more in detail in section 2.

However, the observations do not extend over a sufficiently long time for the third period to be determined accurately; probably it is not shorter than eight times the second period. On account of this uncertainty only part of the analysis outlined above could be performed, namely the determination of one series with arguments ω and τ. This series was computed for the observations reduced and discussed by Dr J. VAN DER BILT [2]), which extend from August 27, 1906 to April 14, 1915, over 3152 days; since this may be a considerable fraction of the third period, it is to be expected that keeping ψ constant will impair the result, especially for the beginning and end of the interval of time covered by the observations.

Yet another partial result has been derived. As has already been mentioned the minimal magnitudes are functions of τ and ψ; since observations of a considerable number of minima, besides those given by VAN DER BILT have been published [3]), it seemed of interest to determine the series representing these functions of τ and ψ, although the results depend on a very uncertain value of the third period.

The treatment of the light-variation of the cluster-type variables RW Draconis and AR Herculis by J. BALÁSZ and L. DETRE [4]) is in many respects

1) Cf. e.g. *Handbuch der Astrophysik*, Bd VI, p. 173, and *Ergänzungsband*, p. 648.

2) This is twice the period $39^{d}{\cdot}267$, given by J. VAN DER BILT, *Recherches Utrecht* VI, p. 93.

3) S. ENEBO, *A. N.* **177**, 313 (1908) mentions a second period of about 3 years.

4) *M.N.* **94**, 856 (1934).

1) *B.A.N.* No. 303 (1937) and No. 306 (1938).

2) *Recherches Utrecht* VI (1916).

3) A. A. NIJLAND, published regularly in the *A.N.*, until **259**, 69 (1936); M. BEYER, *A.N. Astr. Abh.* **8**, C 18 (1930); FR. LAUSE, *A.N.* **244**, 79 (1931) and **251**, 43 (1933); DEAN B. MC LAUGHLIN, *A.J.* **44**, 41 (1934).

4) *Budapest-Svábhegy*, Nr. 5 (1938) and Nr. 8 (1939).

similar to the present analysis. Reference should also be made to a paper by S. OPPENHEIM [1]).

2. *The double series for the observations that have been discussed by* Dr VAN DER BILT.

In this analysis the value 1260 days has been used for the second period.

To start with the observations were divided into 24 groups, corresponding to 24 equal intervals in ω. For each group the magnitude was plotted against the Julian date; all observations were brought in the interval from J.D. 2418200 to J.D. 2419460 by changing, if necessary, the Julian date by 1260 days. Then a graph was drawn for each of the 24 groups, and by means of 24-ordinate analysis coefficients c_j and s_j of $\cos j\tau$ and $\sin j\tau$ ($j \leq 5$) were computed. These coefficients are functions of ω, of which 24 values are known; they have been represented as series in $\cos i\omega$ and $\sin i\omega$ ($i \leq 5$), again by 24-ordinate analysis. Figure 1 shows the coefficients c_0, c_1, s_1, c_2 and s_2 as functions of ω; especially for c_0, s_1 and c_2 the variation with ω is well marked. The higher-order coefficients are small.

The numerical values of the coefficients of the double series are shown in Table 1; the number of decimals given is partly of computational value only. It is seen that a few of the coefficients of the terms with composite arguments, though smaller than those of the terms with simple arguments, are quite significant.

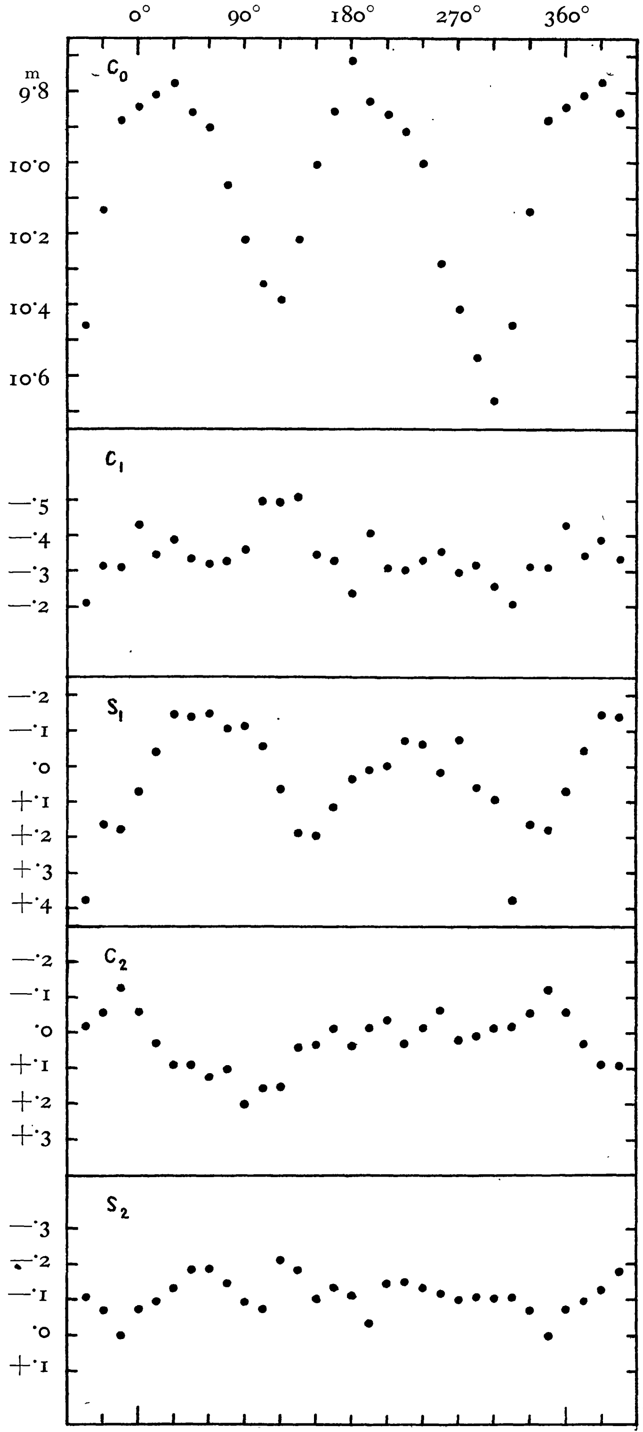
The zero points of ω and τ are respectively at J.D. 2417154·96 and 2418200.

If the argument τ_0 is introduced [2]) by the equation $\tau = \tau_0 + {}^{P_1}\!/_{P_2}\,\omega$, where P_1 and P_2 are the first and second periods respectively, the brightness may be considered as a function of τ_0 and ω. This function is periodic, with period 2π, in τ_0, but not in ω; if ω is decreased by 2π, τ_0 increases with ${}^{P_1}\!/_{P_2} \times 2\pi$. A table has been constructed representing this function, from which light curves may easily be computed by interpolation. Figure 2 shows the function for twelve equidistant values of τ_0; the diagrams exhibit clearly the gradual change in the shape of the light curve.

In the complete solution an argument ψ_0 would occur, analogous to τ_0; it is evident that a number of such tables would have to be constructed, for different values of ψ_0.

The comparison of observed and computed light

FIGURE 1.

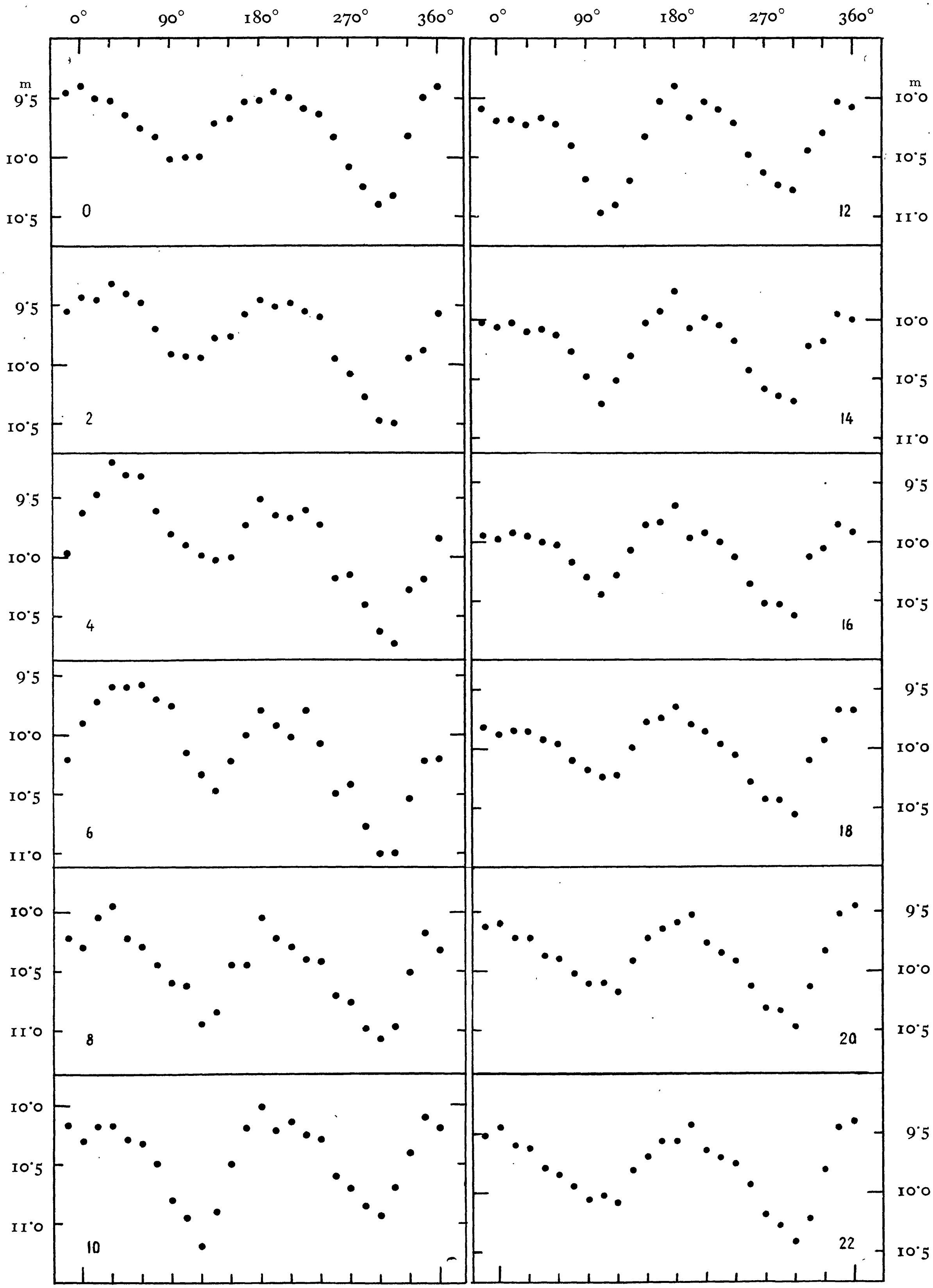


Abscissa: phase ω.

1) *A.N.* **232**, 369 (1928); in connection with a formal analysis of the frequency of sun-spots OPPENHEIM also mentions RV Tauri.

2) Such a transformation of variables is usual in the construction of lunar and planetary tables.

FIGURE 2.



Abscissa: phase ω. The numbers in the lower lefthand, resp. righthand corners of the diagrams are values of τ_0, expressed in units of 15°.

TABLE 1.

Coefficient of cos $(i\omega + j\tau)$

j \ i	0	+1	+2	+3	+4	+5
		m	m	m	m	m
−5		−·001	−·002	+·002	+·004	+·002
−4		+·003	+·011	+·006	+·001	−·004
−3		+·013	+·006	−·013	·000	+·005
−2		−·017	−·033	−·015	−·002	−·004
−1	m	−·018	−·068	−·037	−·015	+·001
0	+10·081	+·033	−·273	−·015	−·027	+·003
+1	−·349	+·035	+·076	−·023	+·012	−·001
+2	+·028	+·006	−·019	+·005	−·001	−·007
+3	+·029	−·007	−·006	−·008	−·008	−·004
+4	−·013	+·005	+·005	+·001	−·003	−·003
+5	+·002	−·002	+·001	−·002	+·001	+·001

Coefficient of sin $(i\omega + j\tau)$

j \ i	0	+1	+2	+3	+4	+5
		m	m	m	m	m
−5		−·001	−·001	−·003	−·001	+·006
−4		·000	−·005	+·006	+·006	−·002
−3		+·003	+·022	+·002	−·005	−·004
−2		+·033	−·004	−·002	−·004	+·009
−1		−·026	−·024	−·007	+·004	+·010
0	m	−·106	−·220	+·003	+·078	+·007
+1	+·023	−·031	+·036	−·005	−·032	+·016
+2	−·118	+·049	+·017	+·002	+·032	+·005
+3	+·032	−·008	+·001	−·001	+·002	+·003
+4	+·007	+·002	−·005	−·001	·000	−·004
+5	−·011	+·001	+·004	−·001	−·003	−·003

curves is satisfactory on the whole, but in the early and late observations there are serious discrepancies. The early observations, near J.D. 2417600, where the variable is very faint, could not be included in drawing the τ-figures; accordingly they lie systematically below the computed curve. In the observations near the end the deep and shallow minima have interchanged and this persistent change is not given by the series. However, as pointed out in section 1, these discrepancies may be ascribed to the fact that ψ has been kept constant.

3. *The series for the minima.*

For each of the two minima a series in τ and ψ has been derived, in an analogous way as described above. Although nothing definitive can be said about the third period, the value of eight times the second period has been used here tentatively; the second period has been taken equal to $1197^{d}{\cdot}64$, a value suiting the observed times of minima minimorum.

The coefficients for the minimum oscillating near $\omega = 120°$ are given in Table 2, those for the minimum oscillating near $\omega = 300°$ in Table 3. No higher multiples than the second, in either τ or ψ, have been considered; $\psi = 0$ for $\pm$ J.D. 2417455.

The remarkable fact that the two minima interchange their depths is very well reproduced by the series.

TABLE 2.

Coefficient of cos $(i\tau + j\psi)$

j \ i	0	+1	+2
		m	m
−2		+·071	−·026
−1	m	+·015	−·011
0	+10·616	−·357	+·066
+1	+·014	−·089	+·051
+2	−·032	−·026	−·037

Coefficient of sin $(i\tau + j\psi)$

j \ i	0	+1	+2
		m	m
−2		+·035	−·00 5
−1		+·022	−·055
0	m	−·171	+·036
+1	−·158	·000	+·059
+2	−·099	+·008	−·007

TABLE 3.

Coefficient of cos $(i\tau + j\psi)$

j \ i	0	+1	+2
		m	m
−2		+·054	+·030
−1	m	−·016	−·005
0	+10·428	−·340	+·050
+1	+·355	−·093	+·035
+2	+·003	−·027	+·043

Coefficient of sin $(i\tau + j\psi)$

j \ i	0	+1	+2
		m	m
−2		+·002	−·007
−1		−·063	+·094
0	m	−·181	+·038
+1	+·155	+·084	−·048
+2	−·022	+·018	·000

Of course the series found here and that derived in section 2 are not independent of each other.

I am indebted to Dr WOLTJER for his help given during this work.

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