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A PHOTOMETRIC INVESTIGATION OF THE SPICULES AND THE STRUCTURE OF THE CHROMOSPHERE

BY L. WOLTJER

The spicules in the solar chromosphere are investigated photometrically on a copy of a plate taken by LYOT with an H_α filter. The plate is calibrated using H_α intensities for the average chromosphere that were determined from observations by DE JAGER. Between $h = 3000$ km and $h = 7000$ km the intensity contrast between some strong spicules and the parts between them is measured. The ratio of the two intensities amounts to a factor 1.9 at 5000 km. The number of H atoms in the second quantum state per cm^3 is found. It appears that this number is 50 times larger in the spicules than in the average chromosphere, if we assume for the average diameter of the spicules the value 1400 km. From counts of spicules of various heights the total number of spicules longer than 8″.7 on the entire solar surface is derived as 30000.

A model of the chromosphere including the spicules is constructed on the basis of a low temperature model by VAN DE HULST. The larger part of the lower chromosphere has a temperature of some 5000 degrees. The spicules have a much higher temperature in our model (Table 5). Between 4000 km and 6000 km the electron temperature is 21000° in the spicules, whereas their electron density drops from $\log N_e = 11.3$ to $\log N_e = 10.9$. In the model the spicules cover 2% of the solar surface and the entire Balmer emission is coming from them. Nearly all electrons are concentrated in the spicules. The density in the spicules is 12 times larger than in the parts between them. A solution is suggested for the discrepancy between the values for N_e derived from the Stark broadening of Balmer lines and those derived by fitting the chromospheric electron gradient to a photospheric model by STRÖMGREN. Some other results of the model are briefly discussed.

1. *Introduction.*

The solar chromosphere, when viewed under very good observing conditions, at an eclipse or through a filter transmitting only the H_α radiation, does not have a smooth outer boundary. Above the mean level of the chromosphere, which extends to some 7″ above the photospheric limb, a great number of so-called spicules protrude.

These spicules were described by ROBERTS [1]), who also studied their lifetimes and movements as observed in H_α. The spicules studied by ROBERTS were situated in the polar regions of the sun. A similar study was made by DIZER [2]), who used films taken by LYOT. MOHLER [3]) estimated the number and the dimensions of the spicules from some plates taken by MARRIOTT at the Swarthmore eclipse expedition. VAN DE HULST [4]) reviewed the observational material and determined the width of a number of spicules on some other plates taken by MARRIOTT. It appears from these studies that the mean lifetime of the spicules is some minutes. The average height measured from the photospheric limb is about 10″, the diameter is between 2″ and 4″. The outward velocity is 20 km/sec or more; the downward movement seems to have the same velocity. The determination of the total number of spicules is difficult, but it will certainly be less than a million and more than ten thousand. Some authors have suggested a correlation between the spicules and the granulation, because of their similarity in lifetime and in dimensions.

The first purpose of the present study is to find quantitative values for the emission of the spicules. Thereafter their physical conditions and relations with the rest of the chromosphere are discussed. A copy of a plate of the chromosphere, selected from a number of plates kindly lent to us by Dr LYOT was investigated photometrically. This copy was a positive reproduction on a process plate. The original plate was taken at the Pic du Midi Observatory on August 22, 1942, on the eastern edge of the sun, with an interference filter, centred on H_α with a bandwidth of 1.5 A. The filter is described by LYOT [5]). On this plate some very conspicuous spicules are visible. In the centre of the plate a prominence is seen, but the chromosphere seems rather quiet. The diameter of the solar image was measured to be 60.2 ± 1.1 cm, so

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[1]) W. O. ROBERTS, *Ap. J.* **101**, 136, 1945.

[2]) M. DIZER, *C. R. Ac. d. Sc.* **235**, 1016, 1953.

[3]) O. MOHLER, *M. N.* **111**, 630, 1951.

[4]) H. C. VAN DE HULST in "The Solar System", Vol. I, Chap V, edited by G. P. KUIPER, Univ. of Chicago Press, 1953.

[5]) B. LYOT, *Ann. d'Astroph.* **7**, 31, 1944.

1 mm on the plate corresponds to 2310 km on the sun. The plate had no calibration marks. Therefore the calibration curve was derived from the known average strength of the H_α emission in the chromosphere. When the calibration curve is known, the true intensity in the spicules can be determined from the photometric readings.

2. *The H_α intensity in the chromosphere.*

The plate was calibrated on the basis of the measurements of the intensity in H_α, made by de Jager 1) at Meudon. These measurements were made with insufficient resolution to show the spicules and thus give the average H_α emission of the chromosphere. De Jager measured the H_α profile up to a height of 8800 km in the chromosphere and on the solar disc between 2000 km and 4000 km from the limb. Adapting Gaussian curves to the measured profiles, the central intensity I_0 and the line width b may be determined. The line width b is defined as the quotient of the total intensity in the line I and the central intensity I_0. The values found for b are:

$3000 \text{ km} < h < 6500 \text{ km} \qquad b = 2.10 \text{ A}.$
$6500 \text{ km} < h < 8000 \text{ km} \qquad b = 1.87 \text{ A}.$

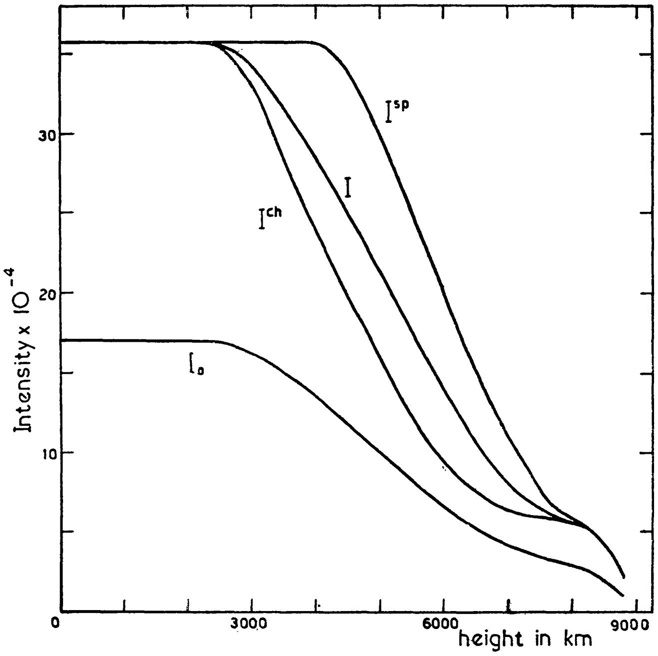
Simple Gaussian curves with these values for their halfwidths represent the measurements of de Jager almost everywhere within 10 %. A comparison with the results of other authors shows, however, that there may be much larger systematic errors.

Cillié and Menzel 2) found at the base of the chromosphere for the halfwidth of H_α the value 2.5 A. Keenan 3) and Unsöld 4) found 2 A. It is remarkable that the line widths measured by Keenan show a marked decrease with height, reaching 1.3 A at 4500 km. Keenan's H_β profile is somewhat broader than that measured by Redman 5) in 1940. Therefore it does not seem impossible that our b's are too large. Some variability in the line width with the solar cycle is not impossible and the irregular structure of the chromosphere suggests that different parts of the chromosphere may give different values at the same time. Theoretically a certain decrease of the line width with height could be expected if, as seems probable, part of the width is caused by selfabsorption (cf. section 6). It is possible, however, that this is compensated by the effects of chromospheric movements 6), or by a positive temperature gradient.

In view of the preceding remarks it seems reasonable to extrapolate the value $b = 2.10$ A to the base of the chromosphere. This extrapolation is necessary, as de Jager's observations at these lower levels were not far enough from the line centre to allow a determination of b.

The central intensities at heights below 3000 km were found by adapting Gaussian curves with this width to the observations. As on the disc no systematic variation with distance from the limb was found for the three profiles between 2000 km and 4000 km, their average was used, and a smooth curve was drawn through the observations. The values obtained for the central intensity I_0 and for the total intensity I are given in Table 1 and Figure 1. The values between + 1000 km and − 2000 km were interpolated.

Figure 1



H_α intensities in the chromosphere measured in erg cm^{-2} sec^{-1} $sterad^{-1}$.

Cillié and Menzel 7) measured at an eclipse the value of $F(h) = \int_h^\infty I\,dh$, that is, the emission per second of a slice of the chromosphere one cm in width and with its lower boundary at a height h. Comparing their values with those obtained from Figure 1, we get:

h	log F (C & M)	log F (Fig. 1)
670 km	(15.36)	14.25
3170 km	13.98	13.95
4000 km	13.49	13.81

The value at 670 km from Cillié and Menzel is extrapolated from other members of the Balmer

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1) C. de Jager, *Rech. Obs. Astr. Utrecht* **13**, 1952.
2) G. G. Cillié and D. H. Menzel, *Harv. Circ.* No. 410, 1935.
3) P. C. Keenan, *Ap. J.* **76**, 134, 1932.
4) A. Unsöld, *Zs. f. Physik* **59**, 353, 1929.
5) R. O. Redman, *M. N.* **102**, 140, 1942.
6) A. Unsöld, *Zs. f. Naturforschung* **7**, 221, 1952.
7) *L.c.*

series. The values at greater heights are in reasonable agreement, although the gradient in the second column seems much steeper. The poor situation in H_α photometry has been pointed out also by VAN DE HULST [1]). Accurate measurements in H_α at an eclipse would be very valuable.

The next problem is to compute what part of the light received is transmitted by the filter. Only the form of the transmission curve is important, the absolute value bringing only a constant in the intensity scale, which cancels out when all intensity values are reduced to their true values. The transmission curve $G(\lambda)$ can to a good approximation be represented by $G(\lambda) = e^{-1.41(\Delta\lambda)^2}$, where $\Delta\lambda$ is the distance from the centre of H_α. The quantity of light transmitted is found by superposition of the line and filter-profile. On the solar disc most of the continuum is absorbed by the filter. The values for I_f, the intensity after transmission by the filter, are given in Table 1. The values between + 500 km and — 2000 km are uncertain, as the H_α profile is not known at these heights.

TABLE 1

H_α intensities in the chromosphere and on the solar disc. I_0 is the central intensity in erg A^{-1} cm^{-2} sec–1 $sterad^{-1}$. I and I_f are the total intensity and the total intensity transmitted by the filter, both in erg cm^{-2} sec^{-1} $sterad^{-1}$. Values between parentheses are interpolated.

height (km)	$I_0 \times 10^{-4}$	$I \times 10^{-4}$	$I_f \times 10^{-4}$
— 4000 } — 2000 }	28.0		99.8
— 1500	(25)		(91.6)
— 1000	(22)		(83.5)
— 500	(19)		(75.4)
500	(17)	(35.7)	(21.6)
1000	(17.0) 1)	(35.7)	(21.6)
2000	17.0	35.7	21.6
3000	16.2	34.0	20.7
4000	13.5	28.4	17.3
5000	10.0	21.0	12.8
6000	6.5	13.7	8.4
7000	4.2	7.9	5.0
8000	3.0	5.6	3.5
8800	1.0	1.9	1.2

1) The observed value was 16.1.

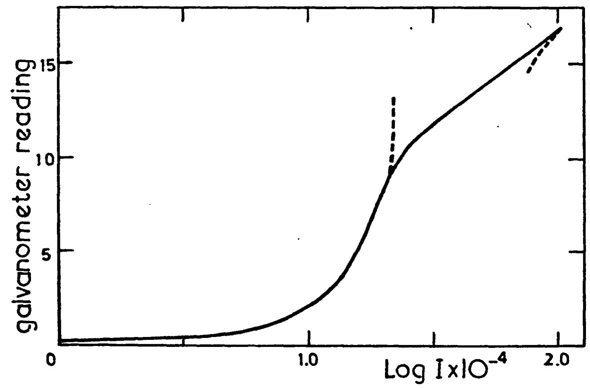
3. *Calibration curve and scattered light.*

One of the plates taken by LYOT was measured with the microphotometer of the Leiden Observatory. Tracings were made parallel to the solar limb over 4°.5 of the solar circumference at seven heights in the chromosphere. For each tracing an average height – because of the curvature of the limb — and an average galvanometer reading was determined. Furthermore some radial tracings extending on to the disc were made. The values for I_f in Table 1 were plotted against the corresponding galvanometer readings in order to obtain a calibration curve (Figure 2). It is immediately apparent that this calibration curve is very irregular between the intensity values $I_f = 20 \times 10^4$ and $I_f = 80 \times 10^4$, corresponding to I_f values found close to the solar limb. Most probably this is caused by light scattered from the photosphere. This effect decreases the intensity on the disc and increases that in the chromosphere compared to the values computed. It is important to make a correction for this effect, as it gives us valuable information on the imperfect definition of the images of the spicules too. Let the intensity distribution in the disc of light caused by a luminous point on the photographic plate be given [2]) by $L = L_0 \frac{a}{\pi} e^{-ar^2}$ where L_0 is the brightness of the point source and r the distance to the centre of the disc. In our case a seems to be rather large, so the diffusion disc will be small. For this reason the chromosphere is covered only by light from the photosphere close to the limb. We can therefore with a sufficiently good approximation consider the photosphere to be of uniform brightness, and the limb as straight. It is then easily found that when the difference in brightness between the chromosphere and the photosphere is L_0, the quantity of scattered light at a height h is given by

$$L = L_0 \frac{a}{\pi} \int_0^{-\infty} \int_{-\infty}^{+\infty} e^{-a\{(h-x)^2 + y^2\}} \, dy \, dx = \tfrac{1}{2} L_0 \{1 - \mathrm{Erf}(h\sqrt{a})\},$$

where Erf denotes the error integral.

FIGURE 2



Calibration curve for the galvanometer readings.
uncorrected curve - - - - - - - -
corrected curve ————

For the uniform brightness of the photosphere the value at $h = -500$ km was taken, for that in the

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[1]) *L.c.*

[2]) A. J. M. WANDERS, *Zs. f. Ap.* **8**, 108, 1934.

chromosphere the value at $h = +500$ km, so L is equal to 53.8×10^4 erg cm^{-2} sec^{-1} sterad^{-1}. The value of a that made the calibration curve smooth was found to be 0.36, where r is measured in units of 1000 km, so the diameter between half intensity points is 2770 km. When r is measured in seconds of arc $a = 0.18$. These values are somewhat uncertain. The final calibration curve is shown in Figure 2.

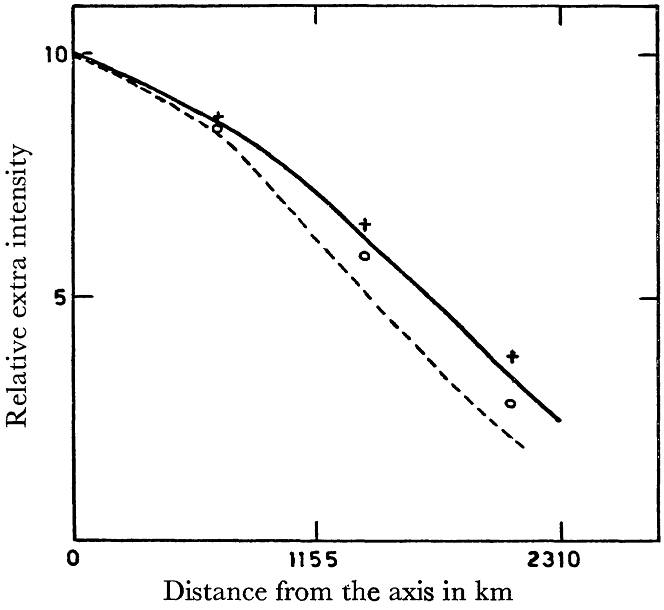
4. *Emission and diameters of the spicules.*

By means of this calibration curve all galvanometer readings were converted into intensity units. On the plate a number of spicules is visible. The mean intensity of seven of these spicules, corrected for diffusion, is denoted by I_f^{sp}, the mean intensity at five points between these spicules by I_f^{ch}; the weighted average of I_f^{sp} and I_f^{ch} is the intensity I_f. All these values are intensities after transmission by the filter. The image on the plate is a projection of the true structure. It is possible that before or behind a visible spicule some other spicules are present, not discernible by their smaller heights. It is possible, too, that there are spicules in the parts between the visible spicules, as only the higher spicules, that protrude above the level of the mean chromosphere, are seen separately.

It will be suggested later that the entire chromospheric H$_\alpha$ emission comes from the spicules, but it is not necessary to take already here such a definite point of view. Thus for the moment we make only a distinction between the visible spicules and the parts between them, as they are seen on the plate. The structure as seen on the plate had to be corrected for diffusion. Considering a spicule with diameter d as a rectangle of uniform brightness and with sides d and ∞, we find, using the scattering function of the preceding section, that the additional brightness at a distance x from the spicule boundary is $I = \frac{1}{2} D_f[\mathrm{Erf}\{\sqrt{a}(d+x)\} - \mathrm{Erf}(\sqrt{a}x)]$, whereas in the centre of the spicule the additional brightness D_f $\mathrm{Erf}(\sqrt{a}d/2)$ remains, where $D_f = I_f^{sp} - I_f^{ch}$.

The value of a was determined in section 3. The measurements of I_f^{ch} were far enough from the spicules to make the corrections to I_f^{ch} very small. From the tracings the mean intensity profile of five spicules at heights 4000 km and 5000 km was determined. Two other spicules were too close to each other for this purpose. For some values of d the intensity profile through the spicules was computed from the equations given above. The best agreement with the observed profile was for $d = 2500$ km, that is 3″.6 (Figure 3). This value is quite uncertain as the profile through the spicule differs not very much from the diffusion profile of a line source. It must be noted that the narrower spicules are registered with a smaller central intensity than the spicules with a larger width, when both have the same real brightness. As the plate that was investigated was chosen because of the clearly discernible spicules, undoubtedly a selection of the larger diameters was made. The mean width of all spicules will therefore be smaller than 3″.6. ROBERTS 1) found for polar spicules 3″ to 4″ in H$_\alpha$. MOHLER 2) estimated on Swarthmore plates as an average value 1″.8, VAN DE HULST 3) 1″ to 2″. I estimated 2″ to 3″ also on Swarthmore plates. Thus the value $d = 2500$ km seems too high, even for the larger spicules and we shall use in the following $d = 2000$ km.

FIGURE 3



The determination of the diameter of the spicules.
Observed intensity profile ———
Computed intensity profile for $d =$ { 3000 km +
2000 km o
0 km - - - -

The measured difference between I_f^{sp} and I_f^{ch} was multiplied by $\{\mathrm{Erf}(\sqrt{a}d/2)\}^{-1} = 1.76$ in order to get the true difference. As I^{ch} needed no correction, I^{sp} was found by summing I^{ch} and the true difference. I^{ch} and I^{sp} were corrected for the transmission through the filter, under the assumption that the line profile is the same for both. They are given in Table 2 and Figure 1. Their difference D is also given in Table 2. The corrections for diffusion in the radial direction are small, as the calibration curve was determined from the undiffused average H$_\alpha$ intensity. So they were omitted.

The values for D are probably correct within 20%, but it is not certain that these rather large spicules form a representative sample for all spicules. It would

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1) *Ap. J.* **101**, 136, 1945.
2) *M. N.* **111**, 630, 1951.
3) "The solar system", Vol. I, Chap. V.

be possible to derive the exact shape of the intensity profile through a circular spicule from the results of section 6 and thus to get a more accurate value for the diameter. The accuracy of the data is too low for such a precise treatment. When we assume a Gaussian curve for the intensity profile, we find that the width of the profile (defined as in section 2) is 2100 km, which is somewhat smaller than our first result.

TABLE 2

H_α intensity in the spicules I^{sp} and in the parts between them I^{ch}. For comparison the value I for the average chromosphere is also given. The difference between I^{sp} and I^{ch} is D. All intensities are given in erg cm^{-2} sec^{-1} sterad^{-1}.

height (km)	$I \times 10^{-4}$	$I^{sp} \times 10^{-4}$	$I^{ch} \times 10^{-4}$	$D \times 10^{-4}$
3000	34.0	35.7[1]	32.5	3.2
4000	28.4	35.7	24.0	11.7
5000	21.0	30.0	16.0	14.0
6000	13.7	20.4	9.4	11.0
7000	7.9	11.0	6.4	4.6

[1] Measured value 37.2

5. *Numbers of spicules and distribution of lengths.*

The number of spicules for different heights was counted on the plate investigated and on two other plates taken by LYOT. The intervals in height were one mm on the plate. This corresponds to 0″.33. Each plate was counted three times. The total number of spicules found on the three plates was 23, 21 and 23 on 10° of the solar limb. As the chromosphere on the second plate seemed less quiet than on the other two, only the latter were used. The frequency distribution thus found is shown in Figure 4*a*. For a comparison we have also plotted the curve found by DIZER[1] from a much larger material. The agreement is reasonable, as his curve comprises quiet and disturbed regions and as it seems from his study that the spicules in disturbed regions are in general somewhat longer than those in quiet regions.

A correction has to be made for the fact that some spicules are hidden behind other ones. The observed height of a spicule is the true length projected against the plane of the sky. Therefore, to obtain a physically useful result, the complete distribution of heights has to be transformed to the true length distribution.

Let us denote by $O(h)\,dh$ the *observed* number of spicules on an arc of the solar limb of unit length and with their heights between $h + \frac{1}{2}dh$ and $h - \frac{1}{2}dh$. Let $Q(h)\,dh$ be the same number in the *complete* height distribution, thus after correction for superposition in the line of sight and let $S(h)$ be the total number of non-overlapping spicules with length greater than h, also on an arc of unit length; thus $S(h) = \int_h^\infty O(h)\,dh$. Let furthermore every spicule have an apparant dia-

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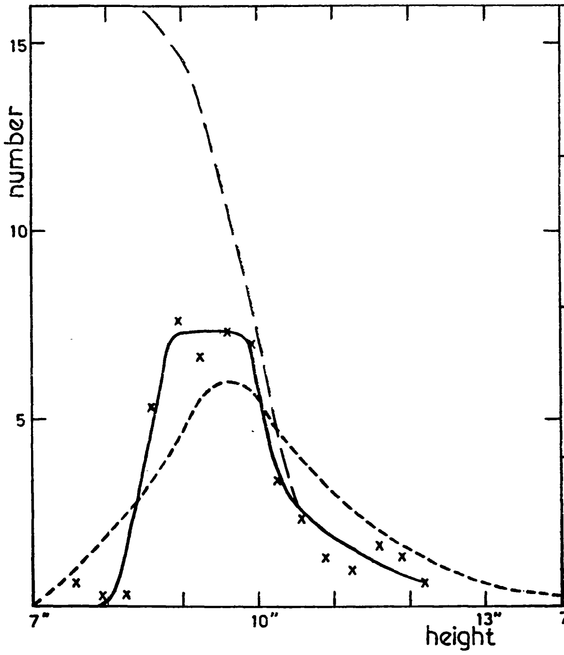
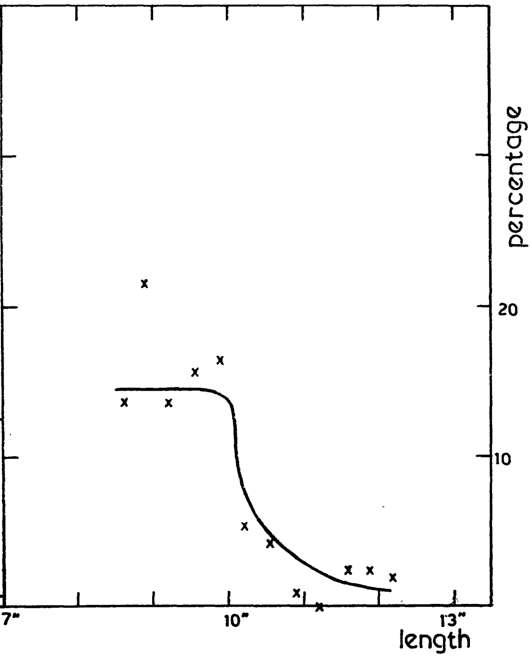
[1] *C. R. Ac. d. Sc.* **235**, 1016, 1953.

FIGURE 4a



Number of spicules of different heights on 20° of the solar limb. Intervals in height 0″.33.
Counted ———, Corrected — — — —, DIZER's curve reduced to the same total number - - - - - -

FIGURE 4b



Percentage of spicules with a certain length. Intervals in length 0″.33.

meter y so that no other spicule with its centre within a distance y on either side of the centre of the first will be separately visible.

So when the visible spicules are far apart, every visible spicule covers effectively an arc of length $2y$ of the solar limb. The lowest spicules are so closely crowded that this is no more the case and the value will be somewhat smaller. But at these heights the spicules are not easy to count and some may be omitted. To correct for this effect we take for all heights the length of arc covered by a visible spicule to be $2y$.

Every spicule with a height h can cover all spicules with height smaller than h. So the fraction of spicules covered with height between $h+\frac{1}{2}dh$ and $h-\frac{1}{2}dh$ is $2yS(h)$, and to find their number per unit length we have to multiply this by $Q(h)\,dh$. Thus the number of spicules counted per unit length and with heights between $h+\frac{1}{2}dh$ and $h-\frac{1}{2}dh$ is given by $O(h)\,dh = Q(h)\,dh\{1-2yS(h)\}$, from which $Q(h)$ can be found (Figure 4*a*). The value for y was estimated as 2000 km.

As the absorption at these greater heights is not very important (cf. section 6), we have obtained now the complete height distribution. It was checked for the spicules investigated photometrically that the height determined visually corresponds to a definite value of D and thus, as the absorption is negligible, to a definite value of the emission from the spicules.

Next we have to transform $Q(h)$ to the true length distribution. Let us denote by $F(l)\,dl$ the number of spicules with lengths between $l+\frac{1}{2}dl$ and $l-\frac{1}{2}dl$ per unit area. We assume all spicules to be directed radially. This seems to be the case for the spicules that are seen on the plates. Then $F(l)\cos\varphi$ is the number of spicule tops in a volume element with volume unity and with its basic plane parallel to the line of sight. Here φ is the angle between the radius to the limb and the radius to the element. As for the spicules that can be seen $\cos\varphi$ differs less than .01 from unity, it may be neglected. It is found now [1]) that

$$Q(h)\,dh = 2\int_{h}^{\infty}\frac{F(l)\{R+l\}}{\sqrt{(R+l)^2-(R+h)^2}}\,dl\,dh \approx \sqrt{2R}\int_{h}^{\infty}\frac{F(l)\,dl}{\sqrt{l-h}}\,dh,$$

as $l << R$ and $h << R$.

This is Abel's integral equation and it was solved numerically (Figure 4*b*). The result is that there is on the average one spicule with length greater than 8″.7 per (14500 km)^2. Under the assumption that this number does not vary over the solar surface this gives 30000 spicules with length larger than 6100 km on the entire surface of the sun. If the average diameter of the spicules is assumed to be 1400 km, 1% of the sun is covered by these spicules. There may be, however, a large number of shorter spicules. On two Swarthmore plates I counted 21 spicules per 10° protruding above the mean level of the chromosphere. This is in good agreement with the values found above. Theoretically it is not yet clear which spectral lines contribute most to the spicules shown on a blue sensitive eclipse photograph. The present observational data give no reason to suspect that the spicules seen in H_α differ greatly from those seen in integrated light.

6. *The number of H atoms in the second quantum state in the spicules.*

The values of I^{sp} and I^{ch} found in section 4 form the basis for computing the emission of the spicules proper. In this computation the optical thickness τ of the chromospheric parts before and behind the spicules is needed.

As there is at present no model of the chromosphere that is sufficiently trustworthy, it was tried to determine τ from the H_α observations. First the optical thickness of the average chromosphere along the line of sight was determined. Next the τ for the regions between the visible spicules (intensity I^{ch}) was determined. From the values of I^{sp} it is possible to derive under certain assumptions the optical depth in the spicule proper and this gives us the number of absorbing H atoms. It is desirable to compare this number with that found for the average chromosphere as most other data on the chromosphere are given for the average chromosphere.

Let B denote the value of the Planck function at the centre of H_α for the excitation temperature T_{exc} and $\varkappa_\lambda$ the absorption coefficient for wavelength λ. Let τ_0 and $\varkappa_0$ denote the values of the optical thickness and the absorption coefficient in the centre of H_α and let n_2 represent the number of H atoms in the second quantum state. Let us use *sp* and *ch* as upper suffixes when we refer to the spicules or the parts between the visible spicules and no suffix if we refer to the average chromosphere. The general expression for the emission in the centre of H_α is $I_0 = B(1-e^{-\tau_0})$. It is seen from this formula that when T_{exc} and thus B is a constant and when $\tau_0 \gg 1$ the emission has a constant value independent of τ_0. Now it is seen from Figure 1 that I_0 is virtually constant for $h < 2500$ km with the value 17×10^4 erg A^{-1} cm^{-2} sec^{-1} sterad^{-1}. As the optical depth most probably increases below 2500 km, as is seen from the gradients of the higher Balmer lines, it is natural to suppose that here I_0 is equal to B and it follows that T_{exc} is 3440°. This is in good

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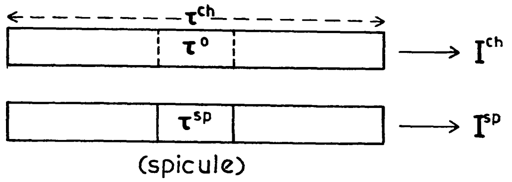
[1]) S. Rosseland, "Theoretical Astrophysics", Oxford Univ. Press, 1936, p. 263.

agreement with the computations of GIOVANELLI who computes [1]) for H_α an excitation temperature between 3150° and 3600° for electron temperatures between 7500° and 25000°, an electron density of some 10^{10} or 10^{11} and a dilution factor for photospheric black-body radiation equal to $\frac{1}{2}$ (cf. section 7). Because of this constancy of T_{exc} it seems reasonable to use for all parts of the chromosphere under discussion the same value for B. From the observed values for I_0 we find the values for τ_0 that are given in Table 4. The values for I_0^{sp} and I_0^{ch} can be found from Table 2, when we assume that the line width for these radiations is the same as for the H_α radiation of the average chromosphere. Let τ° denote the optical depth a spicule would have, when its optical thickness per cm were the same as in the regions that emitt I^{ch}. Then $\tau^{ch} - \tau^\circ$ is the total optical thickness before and behind the spicule, under the assumption that the structure before and behind the spicule is the same as between the visible spicules (Figure 5). We now obtain

FIGURE 5



The optical thicknesses τ^{sp}, τ^{ch} and τ°

$$I_0^{sp} = B\left\{1 - e^{-(\tau_0^{sp} + \tau_0^{ch} - \tau_0^\circ)}\right\},$$

$$I_0^{ch} = B\left\{1 - e^{-\tau_0^{ch}}\right\}.$$

We computed τ_0° from τ_0^{ch} assuming that the spicules were situated just on the solar limb, but the influence of τ_0° is very slight. From I_0^{sp} we found τ_0^{sp}.

The dependence of the line width in I^{sp} and I^{ch} on the optical depth was also considered but it appeared that its neglect made almost no difference in the results. The various values of τ and the emission in the centre of H_α from the spicules proper, E_0^{sp}, which is equal to $B(1-e^{-\tau_0^{sp}})$ are given in Table 4.

The optical thickness τ_0 in a column of hydrogen with uniform density is $k_0 n_2 l$, where l denotes the length of the column. We assume the spicules to have a uniform density perpendicular to their axes. For them l is the diameter, which is equal to 2000 km. Assuming for the average chromosphere an exponential decrease in density with height the total number of H atoms in the second quantum state is equal to $n\sqrt{2\pi RH}$, where H is the scale height, so the equivalent length of the column chromospheric gases depends on the gradient in n_2.

Some arguments will be given in the next section for supposing that the Balmer emission originates mainly in regions with electron temperature higher than 7500°. GIOVANELLI [2]) computed that in this case nearly all the H atoms in the second quantum state are in the substate 2S. So in the absorption process the transition 2S → 3P will take place. The oscillator strength for this transition is $f = .427$ and the absorption coefficient becomes [3])

$$\varkappa_0 = \frac{\pi e^2 \lambda^2}{mc^2 b'} f = \frac{1.62 \times 10^{-13}}{b'},$$

where b' is the line width for $\tau = 0$.
Assuming that $\varkappa_\lambda = \varkappa_0 e^{-a'(\Delta\lambda)^2}$ and thus $\tau_\lambda = \tau_0 e^{-a'(\Delta\lambda)^2}$ the profile of the H_α emission line is given by

$$\frac{1 - e^{-\tau_0 e^{-a'(\Delta\lambda)^2}}}{1 - e^{-\tau_0}},$$

where the central intensity is normalized to unity. Because of the method chosen in section 2 we have to adapt to this profile a curve of the form $e^{-a(\Delta\lambda)^2}$, and to put the result of both formulae equal for a certain value of $\Delta\lambda$, in our case for $\Delta\lambda = 0.8$A. The accuracy is quite good for the part of the line profile where observations were made. A more accurate adaptation is not possible as no measurements were made beyond $\Delta\lambda = 1.0$ A. Now $b' = \sqrt{\frac{\pi}{a'}}$ is the line width the chromosphere would have for $\tau = 0$. The values of b' are given in Table 3. It must be noted that the values given for a certain height are an average over all values above that height, but as the values for b are not known accurately, no corrections for this effect were made. It was found that the exact form of the line profile fits the observations by DE JAGER somewhat better than the simple exponential term.

From the known values of τ_0^{sp}, $\varkappa_0$ and l in the spicules, n_2^{sp} was computed. As the equivalent path length in the chromosphere depends somewhat on the gradient of n_2, two successive approximations were needed to determine n_2. H was found to be 1900 km, so $\sqrt{2\pi RH} = 9.3 \times 10^9$ cm. The n_2's are given in Table 4 together with $c = n_2^{sp}/n_2$. At 7000 km only part of the spicules had an observable emission.

It must be emphasized that the measured spicules were among the largest. On another part of the plate some tracings were made, from which it was possible

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[1]) R. G. GIOVANELLI, *Aust. J. Scient. Res.* **1**, 305, 1948.

[2]) R. G. GIOVANELLI, *Aust. J. Scient. Res.* **1**, 289, 1948.

[3]) A. UNSÖLD, "Physik der Sternatmosphären", Berlin, Springer, 1938.

to determine D, although, as the values of D not corrected for diffusion were less than half those used above, less accurately than in the foregoing. From them we find, taking the diameter of seven spicules as 1500 km, $c=44$ at 5000 km. As the number of very small spicules may be considerable we will assume as the most probable value for the average of all spicules $c=50$ between $h=4000$ km and $h=6000$ km. It is estimated that the error in this result is less than 30%.

TABLE 3
Line widths in the chromosphere for $\tau=0$.

h (km)	b'
3000	1.30 A
4000	1.58 A
5000	1.79 A
6000	1.91 A
7000	1.85 A

TABLE 4
Optical thickness and number of H atoms in the second quantum state per cm^3 for various parts of the chromosphere and the emission in the centre of H$_\alpha$ from the spicules E_o^{sp} in erg A^{-1} cm^{-2} sec^{-1} sterad^{-1}.

h	τ_o	$\log n_2$	τ_o^{ch}	τ_o^o	$E_o^{sp} \times 10^{-4}$	τ_o^{sp}	$\log n_2^{sp}$	c
4000	1.58	3.2	1.11	.03	> 15	> 2	> 5.0	> 56
5000	.89	3.0	.58	.02	12.2	1.27	4.7	66
6000	.49	2.8	.30	.01	7.3	.56	4.4	53
7000	.27	2.5	.22	.01	2.5	.16	3.9	27

7. *A model for the chromosphere.*

The most difficult point in all investigations on the chromosphere at the moment is the determination of the electron temperature T. The line widths of the Balmer lines and the helium lines as well as the low density gradients in the chromosphere can be interpreted in terms of a T of some 20000° or more, although another explanation may be found in the effects of selfabsorption and large-scale mass motions. For it is evident from the spicule phenomenon that the chromosphere is not in a simple hydrostatic equilibrium. Some other features of the Balmer emission point also to a high T. The known facts about the metal emission, the hydrogen abundance and the radio emission in the centimeter waves, on the other hand, indicate a temperature below 7000° or even somewhat less for the lower chromosphere. Therefore it seems rather certain that the larger part of the lower chromosphere has a temperature of the order of 5000°. A survey of the arguments pro and contra a high T was given by WOOLLEY and ALLEN[1]) and by VAN DE HULST[2]).

The spicules are an indication that the chromosphere is not homogeneous. For this reason we shall investigate the possibility of a model characterised by two different temperatures, one for the spicules and one for the parts between them. If the spicules have a high T and not a too large density concentration, almost the entire metal emission and the radio emission will come from the cooler parts between them, as the spicules cover only a small fraction of the solar surface. The Balmer emission is roughly proportional to $\frac{N_e N_p}{T^{3/2}}$. Most of the Balmer emission will come from the spicules because of their more advanced ionization. It was found in the previous section that this is the case for H$_\alpha$. If we should assume the parts between the spicules to be hot we would have difficulties with the radio emission and also with the excitation of H$_\alpha$ (p. 173), which is difficult to explain on the assumption of cool spicules. Qualitative considerations about the division of the chromosphere in columns with different temperatures have been given also by GIOVANELLI[3]) and HAGEN[4]).

VAN DE HULST constructed two models for the lower chromosphere. These models have a spherical symmetry and the temperature is some 6000°. We took these models as our starting point. The models were constructed in the following way. First the metal gradient was determined, from which the hydrogen gradient was found under the assumption of the constant abundance ratio $10^{3.8}$ between hydrogen and the metals. From the heights of the Balmer lines VAN DE HULST found the value of $\frac{N_e N_p}{T^{3/2}}$. Using the Saha equation for hydrogen and obtaining absolute values for N_e and for the number of H atoms per cm^3 N_h by fitting the model for the chromosphere to the models of the photosphere by STRÖMGREN, the values for N_e and T were found separately. In the first model, which we indicate as model I, some corrections were made to the gradients of N_h and N_e that had been derived from the heights of the lines in the flash spectrum, as they were less steep than those derived from the

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[1]) R. v. D. R. WOOLLEY and C. W. ALLEN, *M. N.* **110**, 358, 1950.

[2]) *L.c.*

[3]) R. G. GIOVANELLI, *M. N.* **109**, 298, 1949.

[4]) J. P. HAGEN, *Ap. J.* **113**, 547, 1951.

photometric gradients in the lines as found by Cillié and Menzel[1]). Recent measurements by Lindblad and Kristenson[2]) confirm these steeper gradients and make the discrepancy even somewhat larger. So it seems that the other model constructed by van de Hulst, in which these corrections were omitted, is less correct. Therefore we based the following considerations on the first model only.

Another difficulty is found in the determination of the absolute values for N_e. At 1500 km van de Hulst finds in both models $\log N_e = 10.5$. From the Stark broadening of the higher Balmer lines at 500 km values between 11.0 and 11.5 are found. As the emission observed at 500 km is the integrated emission from higher layers, the values found for N_e should be the values for 1500 km. So the discrepancy is a factor 5 or 10, as was pointed out by van de Hulst. It is important to note that in the model the total number of electrons along a line of sight is determined, whereas the Stark broadening gives immediately the density. A solution for this difficulty may thus be found in the concentration of the electrons in the spicules. Then the electron density in the regions that emit the H lines is larger than in model I, although the total number of electrons along the line of sight need not be larger.

Excitation of the Balmer lines. The new data which we have to incorporate in our model are the values for n_2, derived from the H_α intensities and given in Table 4. We computed values for n_2 from model I, using the Boltzmann equation and obtained the following result.

h	$\log n_2$ (model I)	$\log n_2$ (section 6)
4000 km	2.2	3.2
5000 km	1.8	3.0
6000 km	1.3	2.8

It seems that the agreement between the two columns is poor. However it is very doubtful whether the Boltzmann equation is valid under non-equilibrium conditions. This is also clear from the observed excitation temperature for H_α. If the Boltzmann equation were valid, an excitation temperature of 6000° could be expected whereas the value found in section 6 is 3400°. Giovanelli[3]) has constructed some tables giving the relation between N_e, T and the populations of the first three quantum states of hydrogen under conditions specified by a diluted radiation field corresponding to that of a black body at $T = 5000°$ and various values for the electron temperature and electron density. In entering these tables with the N_e and N_h of model I, we obtain values of the electron temperature of some 20000° and much better fitting values for n_2. This result depends not strongly on the dilution factor for the assumed black-body radiation. However, this cannot be the final model, as the chromosphere has a large opacity in the Lyman continuum for these densities and temperatures. Thus it would be necessary to calculate the influence of the radiation field of the chromosphere itself, which will be much stronger than the diluted temperature radiation at $T = 5000°$. Giovanelli[4]) made such a computation for a special case, but it is difficult to see how the results will be modified in general.

Fortunately the influence of the radiation field of the chromosphere is less serious in the case of our two temperature model. If the spicules have a high T, there will be few unionized H atoms and the Lyman opacity will be small because of the small diameter of the spicules. If in this case the surrounding cooler parts of the chromosphere have a T of 5000°, the conditions for applicability of Giovanelli's tables are fulfilled and it should be expected that the dilution factor for the 5000° black-body radiation is unity, or somewhat less at greater heights. All computations were made using the value 0.5, but the difference between the results from the two values is small. Most of the uncertainty in the present results is caused by the fact that the temperature for the cool parts might be anywhere between 4000° and 7000° and by inaccuracies in Giovanelli's tables. Therefore the results were not recomputed for another dilution factor. Interpolation in the tables was made graphically.

It was found in section 6 that n_2 is 50 times larger in the spicules than in the average chromosphere. If, as suggested, the entire Balmer emission comes from the spicules and thus all H atoms in the second quantum state are to be found in the spicules, 2 per cent of the solar surface should be covered by spicules. This figure is not unreasonable as we found in section 5 that the spicules with length larger than 8″.7 cover 1 per cent of the sun. It is quite possible that the somewhat shorter spicules cover an equal part.

We now divide the chromosphere in three parts: the very low chromosphere is the region below 1000 km, the lower chromosphere that between 1000 km and 6500 km and the upper chromosphere that from 6500 km to 20000 km. In the lower chromosphere we suppose that the entire Balmer emission comes from the spicules. As was seen above from the Stark broadening of the higher Balmer members it seems probable that there is still some spicule structure in the chromosphere at 1500 km. Therefore we took the dividing level with the very low chromosphere, where nothing is known about the structure, at 1000 km. The upper

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[1]) *Harv. Circ.* No. 410, 1935.

[2]) B. Lindblad and H. Kristenson, *Acc. Naz. d. Lin., Atti dei convegni* **11**, Rome 1953, p. 61.

[3]) *Aust. J. Sc. Res.* **1**, 289, 1948.

[4]) *M. N.* **109**, 298, 1949.

chromosphere is more or less homogeneous as only a few of the longer spicules are found in it. This division is of course highly schematic and the transition between lower and upper chromosphere is in reality not sharp.

The lower chromosphere. As we need a definite model for computing purposes, we shall suppose that the temperature between the spicules is 5000° everywhere in the lower chromosphere.

The emission in the Balmer lines that are not very much affected by selfabsorption is proportional to the value of $\frac{N_e\,N_p}{T^{3/2}}$ integrated along the line of sight. Therefore the values of $\frac{N_e\,N_p}{T^{3/2}}$ have to be 50 times larger in the spicules than in the average chromosphere when the entire Balmer emission comes from the spicules. As the ionization is far advanced in the spicules and as the hydrogen abundance is large, we have $N_e = N_p$. The values for $\frac{N_e\,N_p}{T^{3/2}}$ in the average chromosphere are known from model I, so we know $\frac{N_e^2}{T^{3/2}}$ in the spicules. Between 4000 km and 6000 km we also know the values for n_2^{sp} as they are 50 times larger than those for the average chromosphere, which are known from section 6. From $\frac{N_e^2}{T^{3/2}}$ and n_2^{sp} the values for N_h^{sp}, N_e^{sp} and T in the spicules were determined from GIOVANELLI's tables. We found a temperature dropping from 22500° at 4000 km to 19000° at 6000 km, so we took 21000° as an average value. At 7000 km we assumed by extrapolation $T = 21000°$ for the spicules and we found $\log N_e^{sp} = 10.6$ from T and n_2^{sp}.

It appeared that at 4000 km and at 5000 km N_h^{sp} is ten times the value for N_h in the average chromosphere. We assumed tentatively that this remains valid down to 1000 km. N_e^{sp}, T and n_2^{sp} were then determined from the values for N_h and $\frac{N_e^2}{T^{3/2}}$ in the spicules. The values at these lower heights are rather uncertain, as the spicules begin to have a larger opacity in the Lyman continuum.

Between the spicules N_h is known from model I. Assuming $T = 5000°$ we determined N_e and n_2 using the Saha-Boltzmann equation. These values are very uncertain, because of the uncertainty in T. The various results are given in Table 5.

The upper chromosphere. The value for N_e at 7000 km was derived from the heights of Balmer lines, that at 8000 km was found by extrapolation. The values for n_2 at these heights were determined as in section 6. From N_e and n_2 we again find T. At 20000 km we assume $\log N_e = 8.3$ from VAN DE HULST's corona model[1]) and $T = 700000°$. It now appears that the pressure at 20000 km is equal to that at 8000 km. As in a stable model the pressure is not likely to increase outwards it has to be a constant. Thus the value for $N_e T$ in the upper chromosphere is known. This procedure for obtaining $N_e T$ was pointed out by WOOLLEY and ALLEN[2]). At 9000 km the value for n_2 is also known from the H_α intensities, so N_e and T were determined from n_2 and $N_e T$. From the height of the

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[1]) H. C. VAN DE HULST, *B. A. N.* No. **11**, 410, 1950.

[2]) R. v. D. R. WOOLLEY and C. W. ALLEN, *M. N.* **110**, 358, 1950.

TABLE 5

A model for the chromosphere and the spicules.

	Between the spicules				Spicules			
h	$\log N_h$	$\log N_e$	T	$\log n_2$	$\log N_h$	$\log N_e$	T	$\log n_2$
1000	*13.2*	10.3	*5000*	3.6	*14.3*	*11.7*	8000	7.0
2000	*11.3*	9.3	*5000*	1.7	*12.4*	*11.5*	10500	6.3
3000	*10.6*	9.0	*5000*	1.0	*11.7*	*11.4*	12000	6.0
4000	*10.2*	8.8	*5000*	0.6	11.3	*11.3*	21000	*4.9*
5000	*10.0*	8.7	*5000*	0.4	11.1	*11.1*	21000	*4.7*
6000	*9.9*	8.6	*5000*	0.3	10.9	*10.9*	21000	*4.5*
7000	9.9	*9.9*	21000	*2.5*	10.6	10.6	*21000*	*3.9*
8000	9.8	*9.8*	21000	*2.4*				
9000	9.7	*9.7*	25000	*1.8*				
10000	9.5	*9.5*	45000					
12000	9.0	*9.0*	125000					
14000	8.7	*8.7*	250000					
16000	8.5	*8.5*	400000					
18000	8.4	*8.4*	550000					
20000	8.3	*8.3*	*700000*					

The primary data are printed in italics.

H_α line the value of $g\, b_3 \frac{N_e^2}{T^{3/2}}$ at 12000 km was determined, where g is a correction factor to the electron concentrations applied in model I and estimated to have at this height the value 10 (this may be in error by a factor two). The factor b_3 gives the excess of the populations of the third quantum state over that in thermodynamic equilibrium. From GIOVANELLI's tables we find here $b_3 = 7$. At 12000 km there are now two relations between N_e and T, so both can be determined. At the other heights in the upper chromosphere N_e was found by interpolation. T was then found from the value for $N_e T$. The whole model is given in Table 5. The primary data at every height are printed in italics.

It is difficult to estimate the accuracy of the results obtained. For $\log N_e = 10.5$ and $T = 20000°$ an error of 0.1 in $\log N_e$ or 0.2 in $\log n_2$ changes T by 3000°. The value $c = n_2^{sp}/n_2$ is less critical, for a change in this factor changes n_2^{sp} by the same amount and N_e^{sp} by half this, so there is almost no change in T. The table shows an increase in pressure between the spicules from 6000 km to 7000 km. This may not be real but may be due to a somewhat incorrect gradient for N_h in model I or by too high values for N_e at 7000 km. It is also possible that the pressure in the lower chromosphere is larger than the hydrostatic one by the effects of large scale motions.

8. *Conclusions.*

Some features of this model may be noted. The difficulty in the absolute values for N_e is largely removed. At 1500 km we find that we should expect from the Stark broadening $\log N_e = 11.6$. This is even somewhat higher than the observed value; DE JAGER [1]) e.g. obtains $\log N_e = 11.2$. This might be an indication that at lower heights part of the electrons is found between the spicules, but a more accurate discussion of the Stark broadening in the chromosphere may be required [2]).

THOMAS [3]) obtained values for the total number of H atoms in the second quantum state along the line of sight, N_2, from the Balmer decrement. When we compute these values from our model we get the following comparison:

h	$\log N_2$ (THOMAS)	$\log N_2$ (model)
1000 km	15.8	15.2
2000 km	15.2	14.6
3000 km	14.8	14.0

In view of the uncertainties in both procedures the agreement is not unreasonable.

Next the radio emission from our model was computed. The brightness temperature at the centre of the disc is given by $T_b = \int_h^\infty T e^{-\tau_\lambda} d\tau_\lambda$, where τ_λ is the radio optical depth. The measured temperature is the apparent temperature T_a. In the centimeter region, where the limb brightening is slight, $T_a \approx T_b$, whereas above 30 cm it seems that $T_a \approx 2\, T_b$. From our model we computed T_b. The computed values for T_b compare with the observed ones for T_a as follows:

λ (cm)	T_a (obs.)	T_b (comp)
1	8500	10200
3.16	20000	25000
10	50000	43000
31.6	200000	100000

It is seen that the computed values for the shorter wave lengths are somewhat higher than should be expected. The observed values for T_a were taken from data compiled by VAN DE HULST [4]). Better agreement could be obtained without conflict with the values found for n_2 by lowering N_e and T at 7000 km and 8000 km, but then the pressure comes out too low. This may be remedied by giving the base of the corona a somewhat larger height. If we had used the other model by VAN DE HULST for the construction of our model, the computed radio temperatures would have been much higher, so the evidence against this model is strengthened.

ATHAY [5]) determined the temperature at some heights below 2500 km from the Balmer continuum. He obtains $T = 5200°$ at 1000 km and $T = 9120°$ at 2000 km. Thus all evidence indicates that the temperatures in our model and probably at larger heights also the electron concentrations are somewhat too high. This might be explained by a radiation field corresponding to a temperature somewhat higher than 5000°.

We also computed the heat conduction from the corona. We assume with WOOLLEY and ALLEN [6]) that the amount of heat conducted decreases as $\varepsilon = \varepsilon_c \left(\frac{h - h_0}{h_c - h_0}\right)^n$, where ε_c is the amount conducted at the base of the corona, h_0 the level where the heat transport by conduction has finished and h_c the height of the base of the corona. It appears that this formula fits our temperature gradient extremely well. Inserting numerical values we find $n = 3.35$ and $\varepsilon_c = 5.1 \times 10^4$ erg cm^{-2} sec^{-1}.

It seems thus possible to describe the lower chro-

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[1]) *Rech. Obs. Astr. Utrecht* **13**, 1952.

[2]) R. O. REDMAN, *Acc. Naz. d. Lin., Atti dei convegni*, **11**, Rome, 1953, p. 72.

[3]) R. N. THOMAS, *Ap. J.* **111**, 165, 1950.

[4]) H. C. VAN DE HULST, "A Course in Radio Astronomy", Leiden, 1951.

[5]) R. G. ATHAY, *Thesis*, University of Utah, 1953.

[6]) *L. c.*

mosphere as consisting of spicules with a high temperature and pressure and cool parts between them. In view of the fact that the Balmer emission originates mainly in the spicules it seems that investigations based largely on the hydrogen spectrum of the chromosphere, as those by GIOVANELLI [1] [2] and THOMAS [3] refer more particularly to the spicules. The structure in the very low chromosphere is as yet very uncertain. It would be very valuable if data on these layers could be obtained, for these might reveal us something about the dynamics of the spicule phenomenon and its possible connection with the granulae. Perhaps this could be done best by a comparison of the behaviour of metal lines of different excitation potentials. An other way would be photometry outside the centre of H_α, as it was observed by LYOT [4] that the spicule structure becomes more pronounced when the filter is centred somewhat outside the centre of H_α. Most probably this is caused by the fact that the chromospheric absorption is diminished outside the centre of H_α. This might also give some insight in the large width of the H_α line, which is still unexplained.

The heat balance of the chromosphere is a difficult point. It seems probable from the calculations by WOOLLEY and ALLEN [5] that conduction from the corona is sufficient to maintain the temperature distribution in the higher chromosphere above 9000 km. It seems also probable that the spicules furnish the energy in the lower parts, but the exact mechanism is difficult to determine.

My sincere thanks are due to Prof. Dr H. C. VAN DE HULST, who suggested this investigation, for his kind help and valuable criticism during the preparation of this paper, to the late Dr B. LYOT for lending us the plates, to Dr A. DOLFUSS for information about these plates and to Dr C. DE JAGER for giving data on the chromospheric H_α radiation before publication.

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[1] R. G. GIOVANELLI, *Aust. J. Scient. Res.* **1**, 360, 1948.

[2] R. G. GIOVANELLI, *M. N.* **109**, 298, 1949.

[3] R. N. THOMAS, *Ap. J.* **108**, 130, 142; **109**, 480; **110**, 12; **111**, 165; **112**, 337, 343; **115**, 550; 1948–1951.

[4] *Ann. d'Astroph.* **7**, 31, 1944.

[5] R. v. D. R. WOOLLEY and C. W. ALLEN, *M.N.* **110**, 358, 1950.

Note added in proof. The suggestion made above, that the temperatures in our model (Table 5) may be somewhat too high, is supported by the new data given in PIDDINGTON's recent article on the solar radio emission [1]. As a tentative correction we may assume $T = 10000°$ at 7000 km. We still assume that hydrogen is ionized, although the temperature is somewhat too low. This will probably be the case if there is a radiation field corresponding to a black-body temperature higher than 5000°. We lower in the upper chromosphere all temperatures between 6500 km and 15000 km in the same ratio and we diminish log N_e in this height interval by 0.24. This modified model gives the following values for T_b, which may be compared with values for the non-sunspot component given by PIDDINGTON.

λ (cm)	T_b (comp.)	T_b(obs.)
1	6900°	7100°
3.16	12200	12000
10	20500	22000
31.6	48000	54000

The pressure gradient may be made stable by shifting the base of the corona to a somewhat larger height.

The conclusions about the spicules and the temperatures below 6500 km are not directly affected by this change, but the indications are that the temperature in the spicules may be somewhat lower and the temperatures between the spicules somewhat higher than the values in Table 5.

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[1] J. H. PIDDINGTON, *Ap. J.* **119**, 531, 1954.